

# So what's wrong with the Parcel View?

adiabatic view

microphysics – relaxation times – saturation adjustment

microphysical coupling to dynamics

clustering – rain processes – sedimentation

cloud partition – core vs. margins

mixing - entrainment

thermodynamic feedbacks

radiation feedbacks

# All about cloud microphysics

Key:

$r$  = radius in micrometers

$n$  = number per liter

$V$  = terminal velocity in centimeters per second

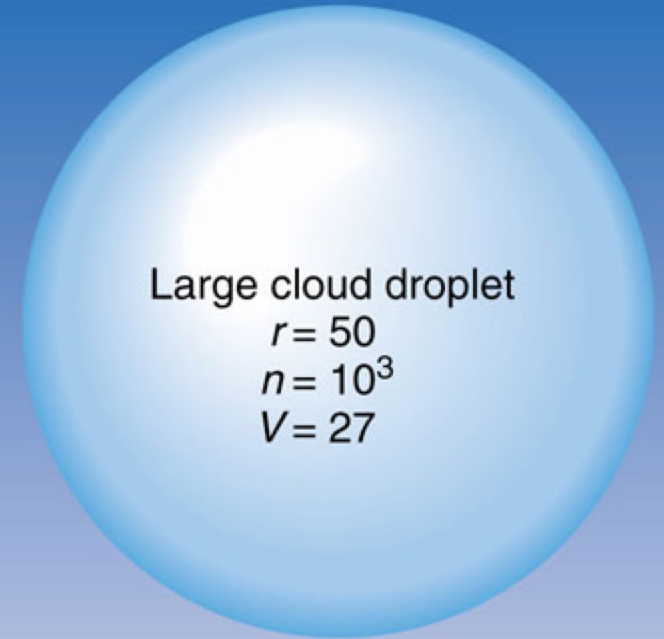


Typical cloud droplet

$r = 10$

$n = 10^6$

$V = 1$



Large cloud droplet

$r = 50$

$n = 10^3$

$V = 27$

• Typical condensation nucleus

$r = 0.1$

$n = 10^6$

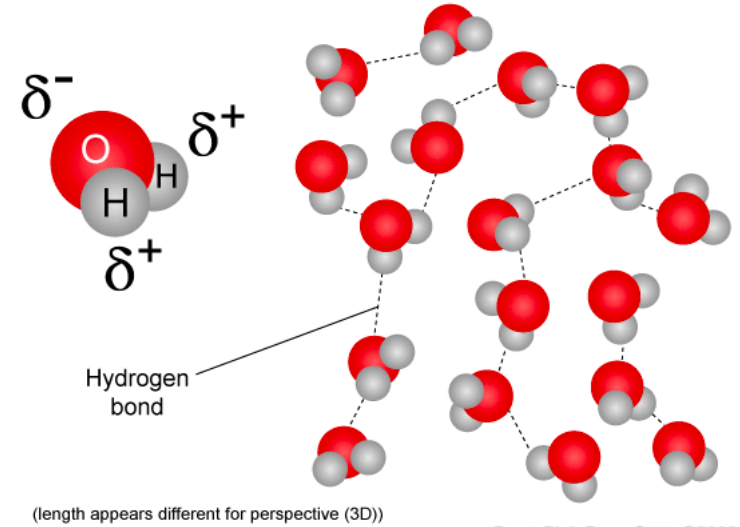
$V = 0.0001$

Typical raindrop  $r = 1000$ ,  $n = 1$ ,  $V = 650$

# Nucleation



From Wikipedia - Michael Apel



Dept. Biol. Penn State ©2002

## Formation and destruction of water embryo clusters

# Kelvin's Equation

Homogeneous nucleation, i.e. clustering of water molecules. Such clustering requires work  $A\sigma$  to create the surface area  $A$ .  $\sigma$  is work needed to create water cluster-air per unit area. It is equal to the **surface tension**.

The total energy required to create cluster with volume  $V$  and surface area  $A$  is:

$$\Delta E = A\sigma - nVkt \ln \frac{e}{e_s}$$

$n$  is the number of water molecules per unit volume of liquid and  $k$  is the Boltzmann constant. In the clusters radius ( $R$ ) view

$$\Delta E = 4\pi R^2\sigma - \frac{4}{3}\pi R^3 nkt \ln \frac{e}{e_s}$$

$\Delta E < 0$  to create a stable cluster in which the molecules “prefer” the liquid phase

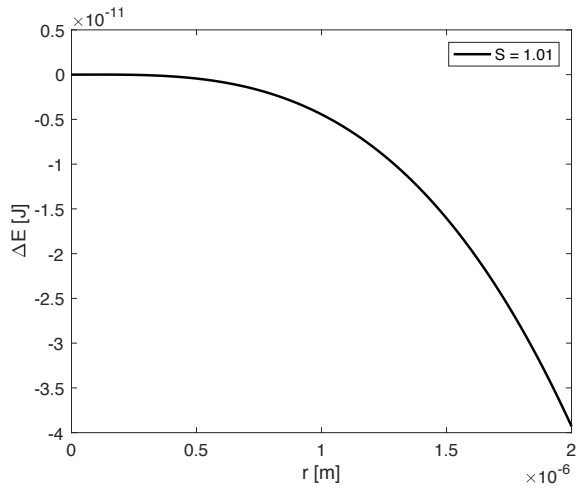
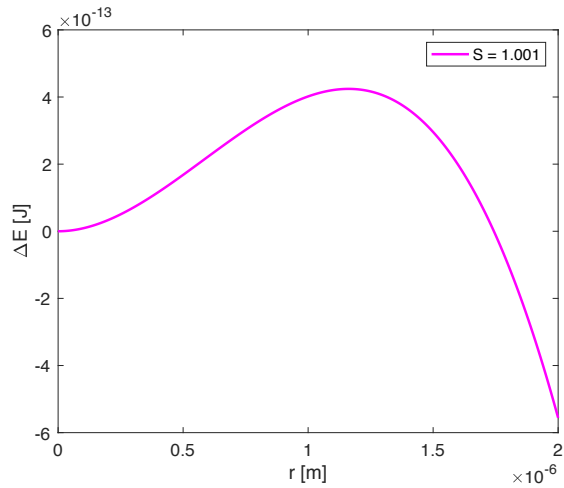
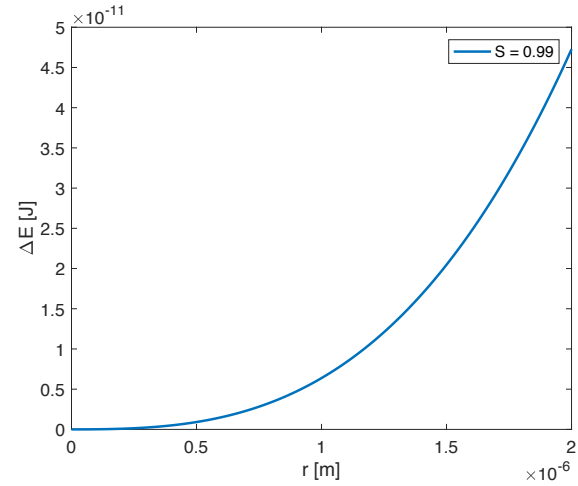
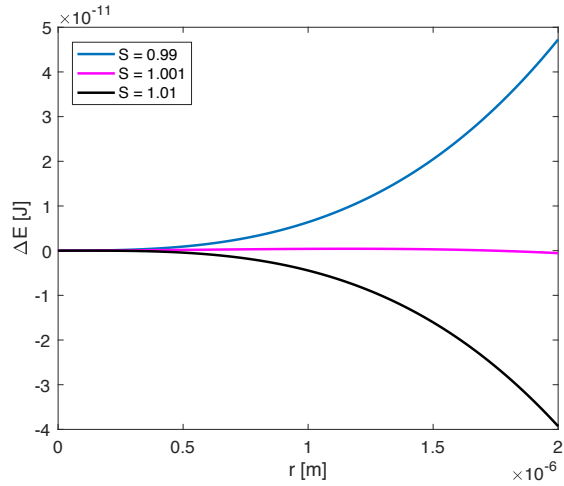
$$\sigma = 7.5 * 10^{-2} N/m$$

$$\text{Boltzmann's const } k = 1.380649 \times 10^{-23} \text{ J/k}$$

$$\text{Number of water molecules in } m^3 = 3.3424e+28$$



$$\Delta E = 4\pi R^2 \sigma - \frac{4}{3}\pi R^3 nkT \ln \frac{e}{e_s}$$



The  $\frac{\partial E}{\partial R} = 0$  point finds the critical  $R=r$  for nucleation

$$r = \frac{2\sigma}{nkT \ln \frac{e}{e_s}}$$

$r$  is an unstable fixed point.

For a given  $r$  we can calculate the required  $e$  to allow  $r$  to be unstable fixed point

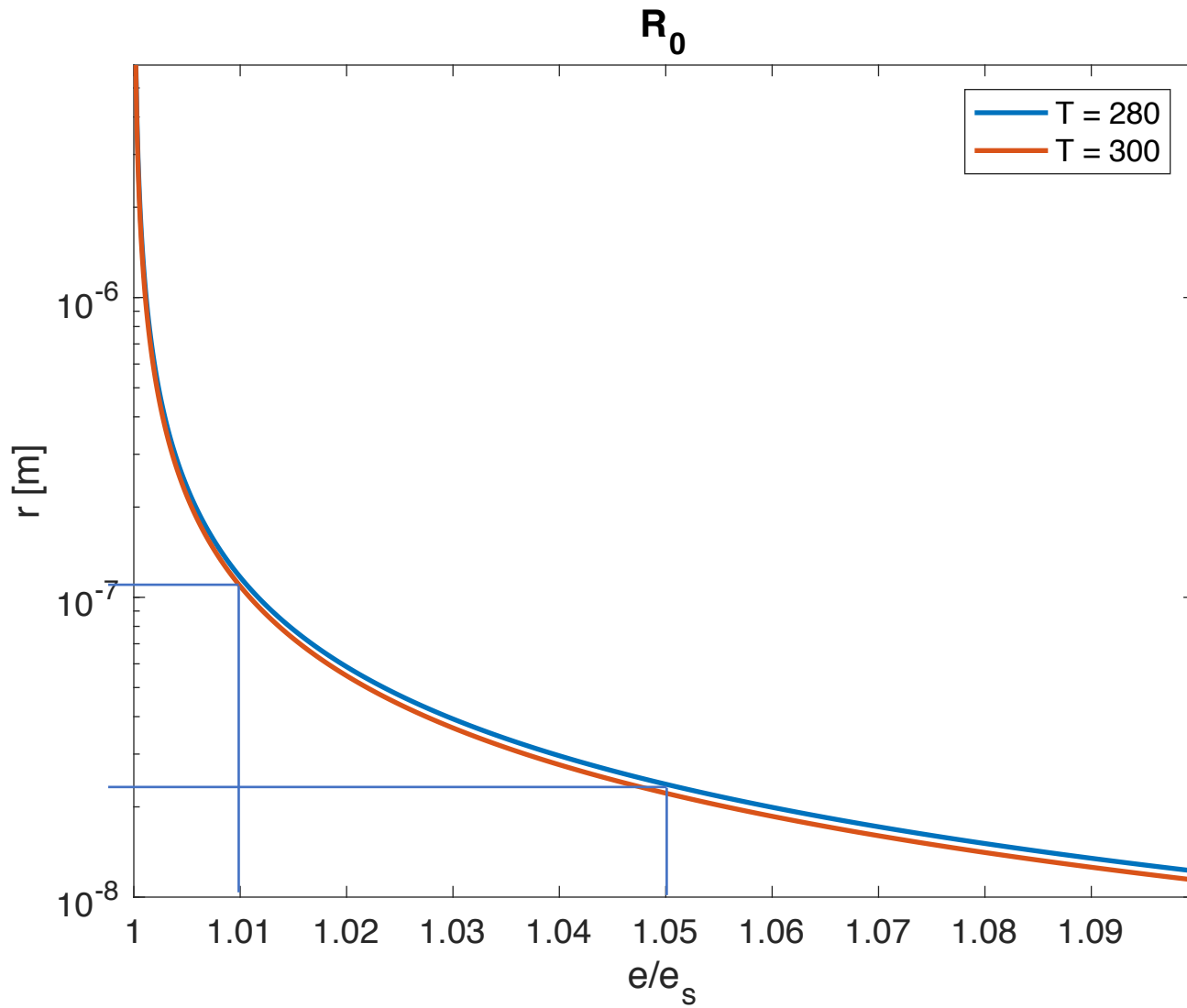
$$e = e_s e^{\frac{2\sigma}{nkTr}}$$

Back to the eq. of state  $k = \frac{R^*}{N_A}$

$$e = e_s e^{\frac{2\sigma}{R_v \rho_{liq} T r}}$$

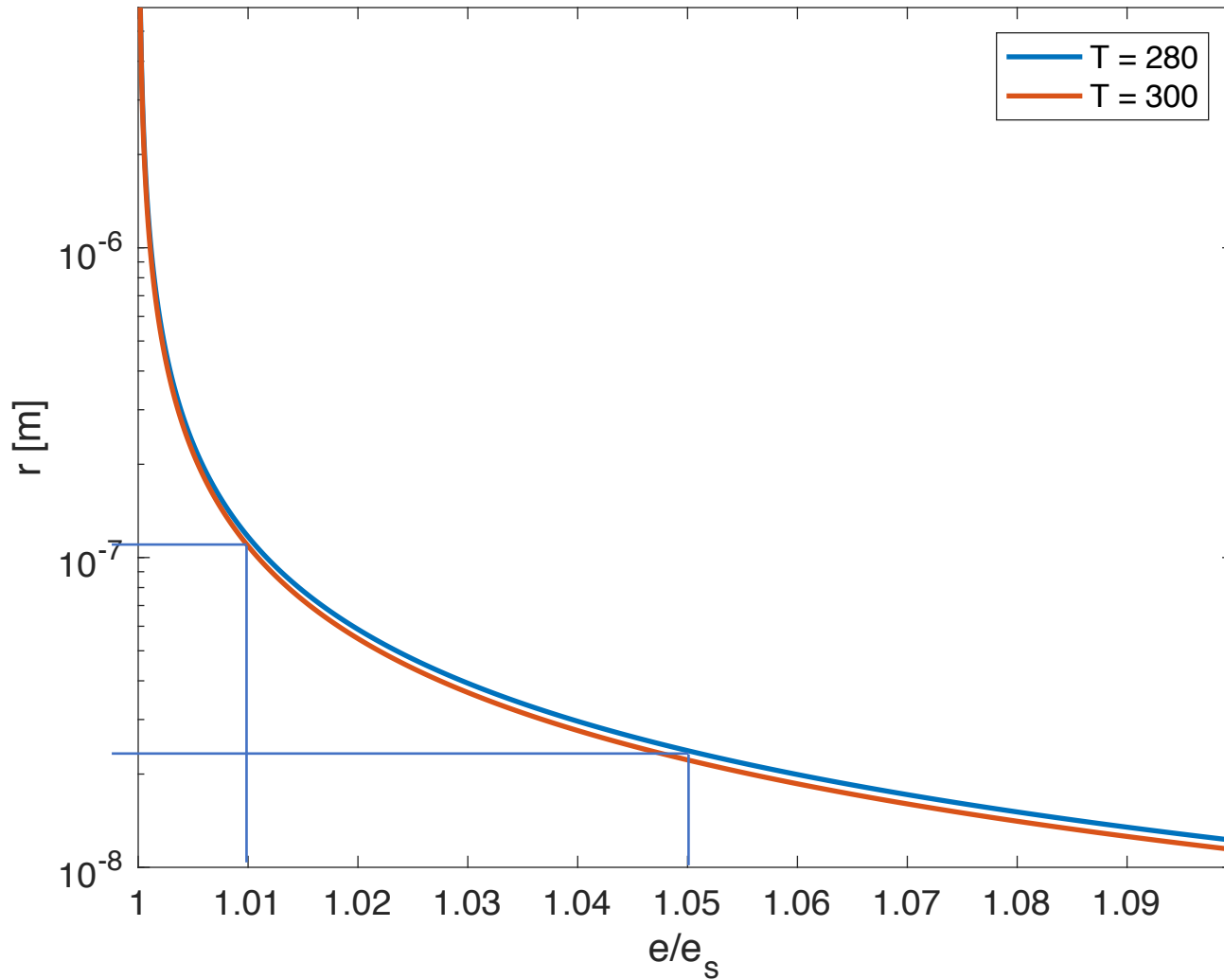
$$e = e_s e^{\frac{2\sigma}{R_v \rho_{liq} T r}}$$

$$\frac{e}{e_s} = \exp\left(\frac{2\sigma}{R_v \rho_{liq} T r}\right)$$



$S = 0$       $r = \infty$   
 $S = 1\%$      $r = 0.12 \mu$      $2.5 \times 10^8$   
 $S = 5\%$      $r = 0.02 \mu$      $2.8 \times 10^5$

$R_0$



## Contribution of a non-volatile solute - Raoult's Law

Over a plane surface of water, the reduction in vapor pressure is proportional to the mass of the solute ( $m$ ), the degree of ionic dissociation ( $i$ ), and inversely to the solute molecular weight ( $M_s$ )

$$\frac{e_{sol}}{e_{\infty}} = 1 - \frac{3imM_w}{4\pi M_s \rho_w r^3} = 1 - \frac{b}{r^3}$$

When combining the curvature (Kelvin's) effect:

$$\frac{e_{s,r}}{e_{\infty}} = \left(1 - \frac{b}{r^3}\right) e^{\left(\frac{2\sigma}{R_v \rho_{liq} T r}\right)}$$

Köhler equation:

$$\frac{e_{s,r}}{e_\infty} = \left(1 - \frac{b}{r^3}\right) e^{\left(\frac{2\sigma}{R_v \rho_{liq} T r}\right)} = a$$

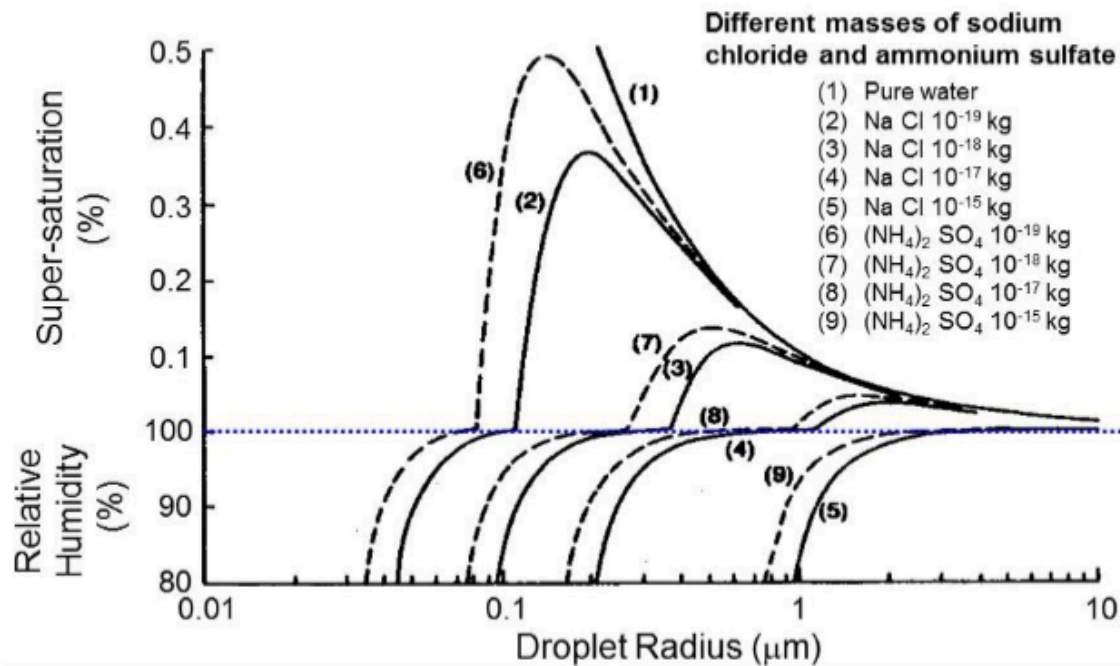
$$\frac{e_{s,r}}{e_\infty} = \left(1 - \frac{b}{r^3}\right) e^{\left(\frac{a}{r}\right)}$$

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$$\frac{e_{s,r}}{e_\infty} \approx 1 + \frac{a}{r} - \frac{b}{r^3}$$

Approximation works for r not too small

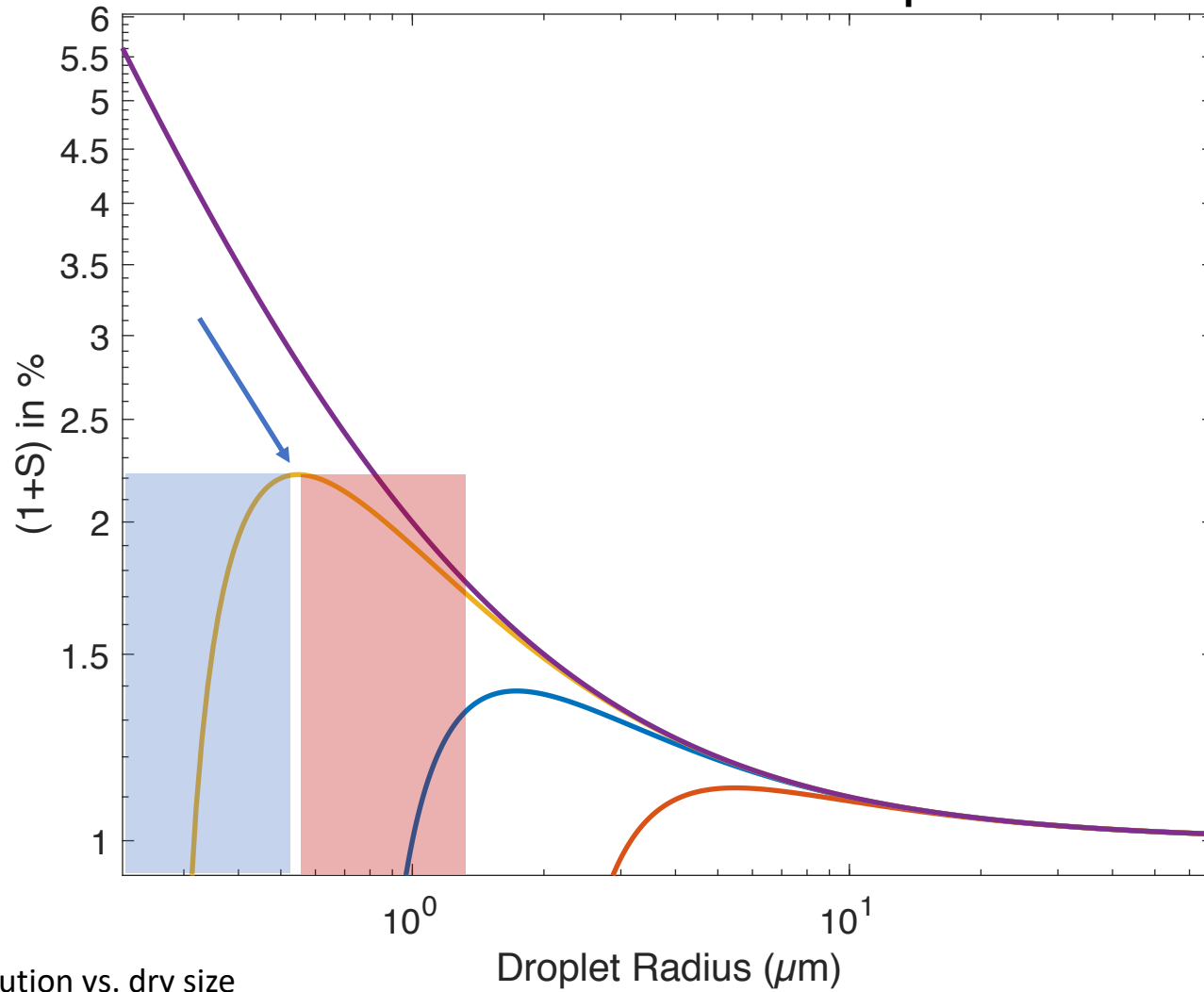
Köhler, H., 1936. The nucleus in and the growth of hygroscopic droplets. Trans. Faraday Soc., 32, 1152–1161.



From <https://www.chegg.com/homework-help/questions-and-answers/use-kohler-curve-plots-answer-following-question-consider-01-droplet-pure-water-environment-q20381421>

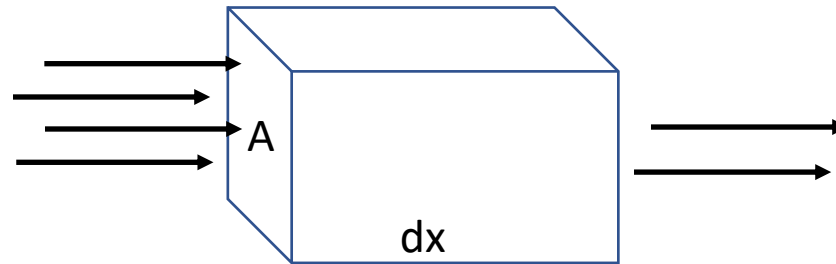


## Kohler Curves for Solution Droplets



Talk about: solution vs. dry size  
Stable vs nonstable fixed points

A short detour – talking about gradients fluxes and diffusion



Diffusion is the outcome of random walk.

So why do we see net transport if the walk is random?

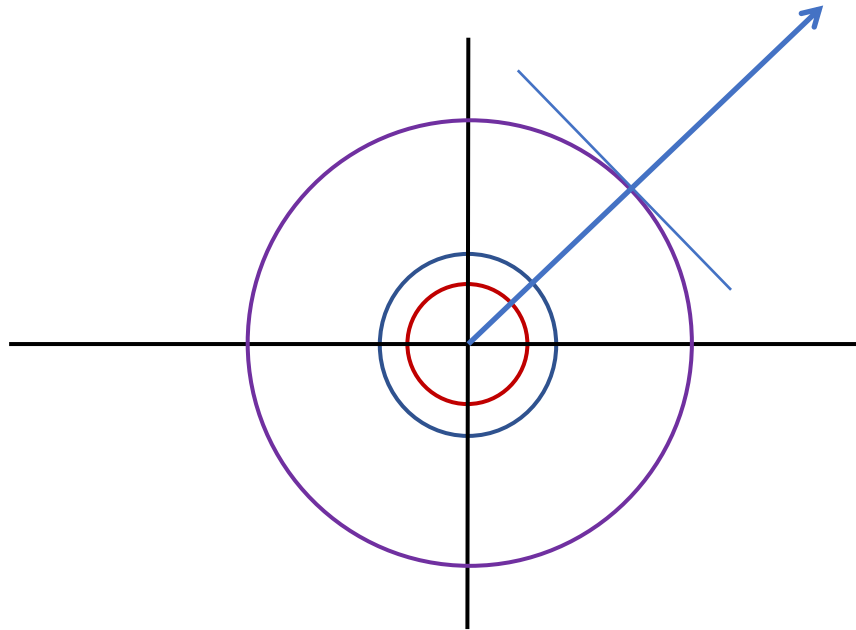
Gradients as the fuel of diffusion. Newton's coffee (cooling rate is proportional to the T differences). Gradient driven.

## A single drop point of view

Assume that we have a water vapor sink located at the origin of polar coordinates space. Assuming no averaged local accumulation of vapors in the ambient.

This implies that the integrated flux crossing any concentric sphere should be constant.

This also implies no average gradients along the orthogonal to the radius plain.



$$\frac{dM}{dt} = 4\pi r D (\rho_v(\infty) - \rho_v(r))$$

Since  $M = \rho_l \frac{4}{3} \pi r^3$  and using the ideal gas for vapor assuming no gradient in T yields:

$$r \frac{dr}{dt} = \frac{D \rho_v(\infty)}{\rho_l} \frac{(e_v(\infty) - e_v(r))}{e_v(\infty)}$$

We have neglected L release creating T gradients and we will assume that the droplet is large enough so the curvature effect is negligible. We can assume that on the surface of the droplet  $e_v(r) = e_s(T)$

We define supersaturation as:

$$S = \frac{(e_v(\infty) - e_s)}{e_v(\infty)} \cong \frac{(e_v(\infty) - e_s)}{e_s}$$

and therefore, the droplet growth eq. becomes

$$r \frac{dr}{dt} = \frac{D \rho_v(\infty)}{\rho_l} S$$

For the simplistic assumptions for which the droplets do not consume S and other than r all variables are kept constant  $r \frac{dr}{dt} = \frac{D\rho_v(\infty)}{\rho_l} S$  can be solved analytically

$$r = \sqrt{2at + r_0^2}$$

where  $a = \frac{D\rho_v(\infty)}{\rho_l} S$

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$$r = \sqrt{2at + r_0^2}$$

$$\text{where } a = \frac{D\rho_v(\infty)}{\rho_l} S$$

But in reality S is not constant. It changes in space and in time. It has sources (cooling) and it has sinks (condensation, mixing)

$$\frac{dS}{dt} = S_{rc} - S_{nk}$$

$$\frac{dS}{dt} = Q_1 \frac{dz}{dt} - Q_2 \frac{d\chi}{dt}$$

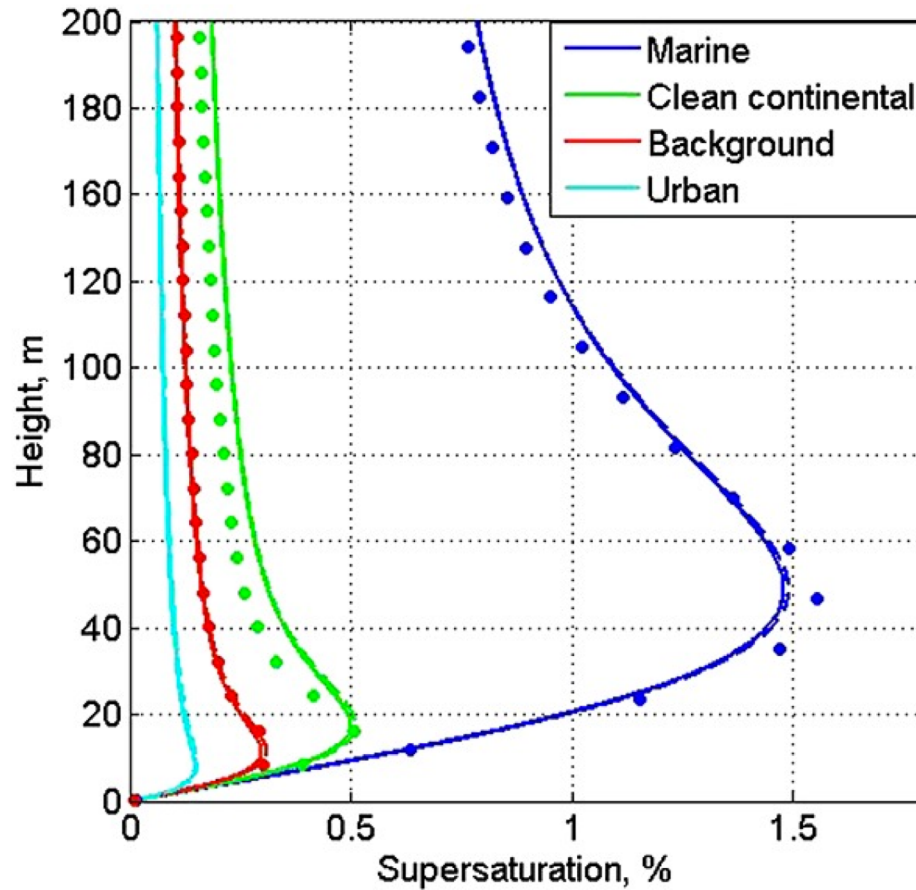
$$\frac{dS}{dt} = S_{rc} - S_{nk}$$

$$\frac{dS}{dt} = Q_1 \frac{dz}{dt} - Q_2 \frac{d\chi}{dt}$$

Where  $Q_1 \frac{dz}{dt}$  the source term is proportional to the updraft (cooling rate) and  $Q_2 \frac{d\chi}{dt}$  the sink term is proportional to the rate of condensation  $\frac{d\chi}{dt}$  per unit mass of air.

Solving the coupled equations of  $\frac{dS}{dt}$  and  $\frac{dr}{dt}$  imply a maxima in the S near cloud base.

Solving the coupled ODE's yields a maxima in the S profile:



Talk about:

Pinsky, et al, 2014

Run away effect – size distribution separation

Haze

Activation range within the cloud



# Aerosol effect on clouds I

Immediate feedbacks of changes in the droplet size distribution in the cloud scale:

- (1) Cloud's optical properties
- (2) Droplet collection processes
- (3) Droplets Mobility
- (4) Diffusion processes efficiency
- (5) Twilight – mixing – entrainment

All of the above link cloud microphysics to dynamics

The activation and growth by diffusion part already imply strong aerosol control

