Atmospheric Aerosol Physics, Physical Measurements, and Sampling

Particle Diameters & Size Distribution

São Paulo School of Advanced Science on Atmospheric Aerosols: properties, measurements, modeling, and effects on climate and health

Member of Leibniz Association

ECAC European Center for Aerosol Calibration

WCCAP World Calibration Centre for Aerosol Physics

TROPOS Leibniz Institute for Tropospheric Research

Mobility or Stokes Diameter

Particle Diameter

The particle size is defined either by the diameter, D_P

The diameter is typically given in µm [10^{-6} m] or nm [10^{-9} m]

The diameter is normally defined as equivalent diameter (non-spherical particles are described as spheres).

Particle diameter definitions

Different measurement principals result in different definitions for the particle diameter:

- Stokes (mobility) diameter, $D_{P,St}$
- Optical diameter, $D_{P,Opt}$
- Aerodynamic diameter, $D_{P,Ae}$

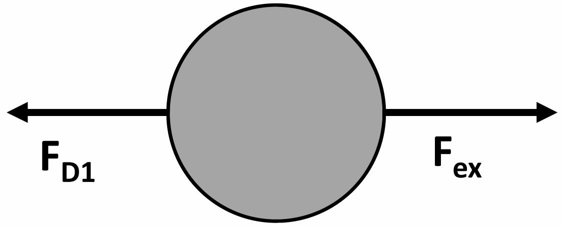
Important is also the volume equivalent diameter, $D_{P,Ve}$

Stokes Diameter (Mobility Diameter)

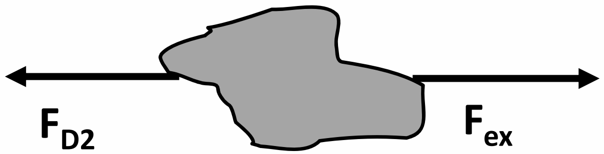
The Stokes diameter is defined for a **uniform particle motion**, where the external force equals the drag force. The motion is independent of the particle density.

$$\vec{u}_P = \text{constant}$$

$$\vec{F}_{ex} = \vec{F}_D = \frac{3\pi \cdot \eta \cdot \vec{u}_P \cdot D_P}{C_C}$$



- For a spherical particle, the Stokes diameter $D_{P1,St}$ can be determined then by the drag force and the particle velocity.
- In case of a spherical particle, the Stokes diameter is equal the geometric diameter and the volume equivalent diameter $D_{P1,Ve}$.

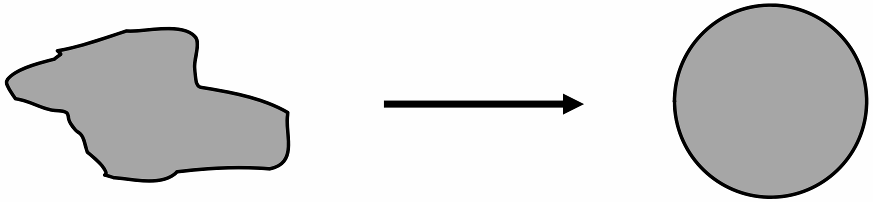


For an irregular particle, the Stokes diameter, if $D_{P2,St} = D_{P1,St}$

$$\vec{u}_{P2} / \vec{F}_{D2} = B_2 = B_1 = \vec{u}_{P1} / \vec{F}_{D1}$$

The volume equivalent diameter of an irregular particle can be calculated by knowing additionally the dynamic shape factor.

$$\vec{F}_{D} = \frac{3\pi \cdot \eta \cdot \vec{u}_{P} \cdot D_{P,Ve}}{C_C} \cdot \chi$$

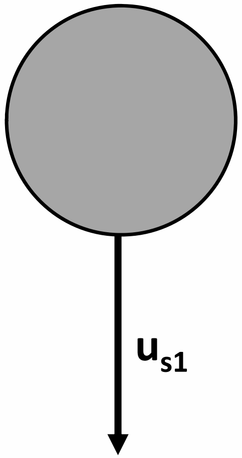


The volume equivalent diameter $D_{P2,Ve} < D_{P1,Ve}$

Aerodynamic Diameter

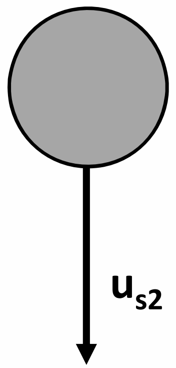
Aerodynamic Diameter

The aerodynamic particle diameter is used when the drag force depends on the particle density such as for sedimentation velocity.



For a certain sedimentation velocity, the aerodynamic particle diameter can be calculated by using $\rho_P = \rho_0 = 1$

$$\vec{u}_s = \frac{\rho_P \cdot D_P^2 \cdot C_C \cdot \vec{g}}{18\eta} = \frac{\rho_o \cdot D_{P,Ae}^2 \cdot C_C \cdot \vec{g}}{18\eta}$$

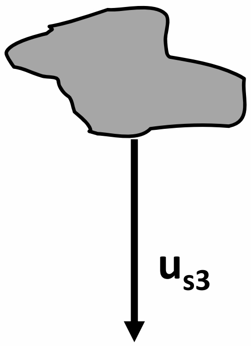


If the particle density is $\rho_\mathrm{P} \neq 1$ and the sedimentation velocities are $\vec{u}_\mathrm{S2} = \vec{u}_\mathrm{S1}$ the aerodynamic diameters are identical $D_\mathrm{P2,Ae} = D_\mathrm{P1,Ae}$

The Stokes diameter can be calculated from the aerodynamic diameter, if the particle density is known.

For a spherical particle with the density $\rho_\mathrm{P} > 1$, the Stokes (volume equivalent) diameter is smaller than the aerodynamic particle diameter and can be calculated to:

$$D_\mathrm{P2,St} = D_\mathrm{P2,Ve} = D_\mathrm{P1,Ae}\sqrt{\frac{\rho_0}{\rho_\mathrm{P}}}$$



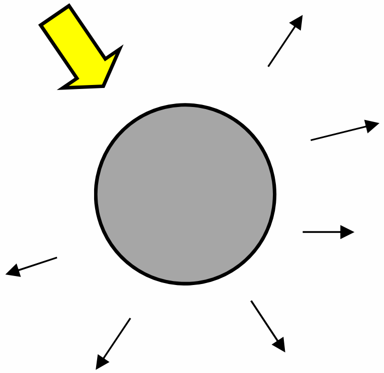
If the particle density is $\rho_{\mathrm{P}} > 1$, the particle is irregular $\chi > 1$, and the sedimentation velocities are $\vec{u}_{\mathrm{S3}} = \vec{u}_{\mathrm{S1}}$, the aerodynamic diameters are identical $D_{\mathrm{P3,Ae}} = D_{\mathrm{P1,Ae}}$.

$$D_{\mathrm{P3,Ve}} = D_{\mathrm{P1,Ae}} \sqrt{\frac{\rho_0}{\rho_{\mathrm{P}}} \chi}$$

Optical Diameter

Optical Diameter

The optical particle diameter is based on the particle scattering σ_P, if the particle is illuminated.

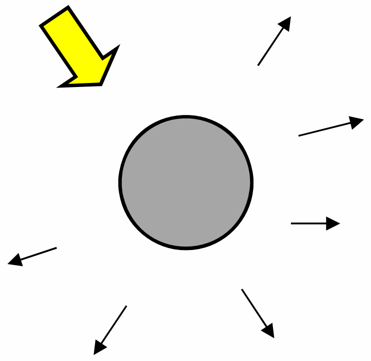


σ_P

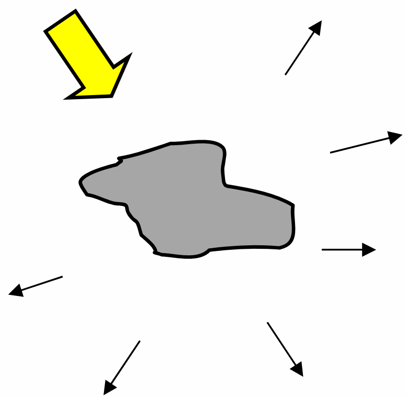
An illuminated spherical particle with a known Stokes diameter (latex particle)and a known refractive index gives a certain particle scattering

The optical diameter of this particle is then calibrated to:

$$D_{P1,Latex} = D_{P1,Opt} = D_{P1,St}$$



An illuminated spherical particle with unknown size and refractive index, but with the same particle scattering $\sigma_{P2} = \sigma_{P1}$ has the same optical diameter $D_{P2,Opt} = D_{P1,Opt}$.



An illuminated irregular particle with unknown shape and refractive index, but with the same particle scattering $\sigma_{P3} = \sigma_{P1}$ has the same optical diameter $D_{P3,Opt} = D_{P1,Opt}$.

Vaccuum Aerodynamic Diameter

Vacuum Aerodynamic Diameter

In the free-molecular regime (vacuum), the aerodynamic diameter is called the vacuum aerodynamic diameter

$$\frac{\rho_P \cdot D_P^2 \cdot C_C \cdot \vec{g}}{18\eta} = \frac{\rho_o \cdot D_{P,VAe}^2 \cdot C_C \cdot \vec{g}}{18\eta}$$

The Cunningham Slip Correction factor for the free molecular regime can be simplified to:

$$C_C \approx \frac{\lambda}{D_P}$$

$$\frac{\rho_P \cdot D_P \cdot \vec{g}}{18\eta \cdot \lambda} = \frac{\rho_o \cdot D_{P,VAe} \cdot \vec{g}}{18\eta \cdot \lambda}$$

The volume equivalent diameter can then be determined by:

$$D_{P,Ve} = D_{P,VAe} \frac{\rho_0}{\rho_P}$$

Particle Number Size Distribution

Particle Number Size Distributions

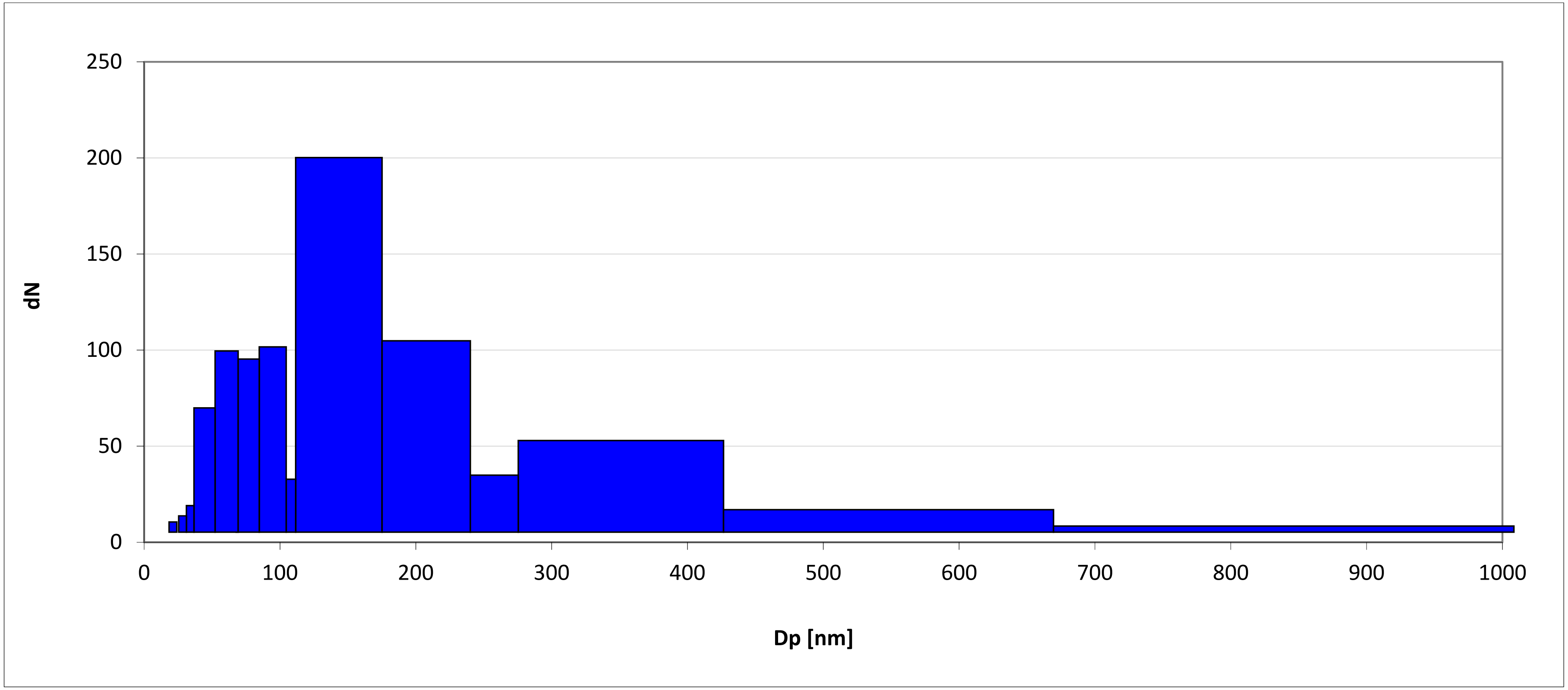
- It would probably be the best to display an aerosol size distribution as a list of concentrations of more than 1000 particle sizes classes.
- However, aerosol instruments normally provide only a limited number of size intervals and concentrations.

Example

An instrument with 10 size classes provides 21 values (upper and lower diameters, and concentrations for each size class).

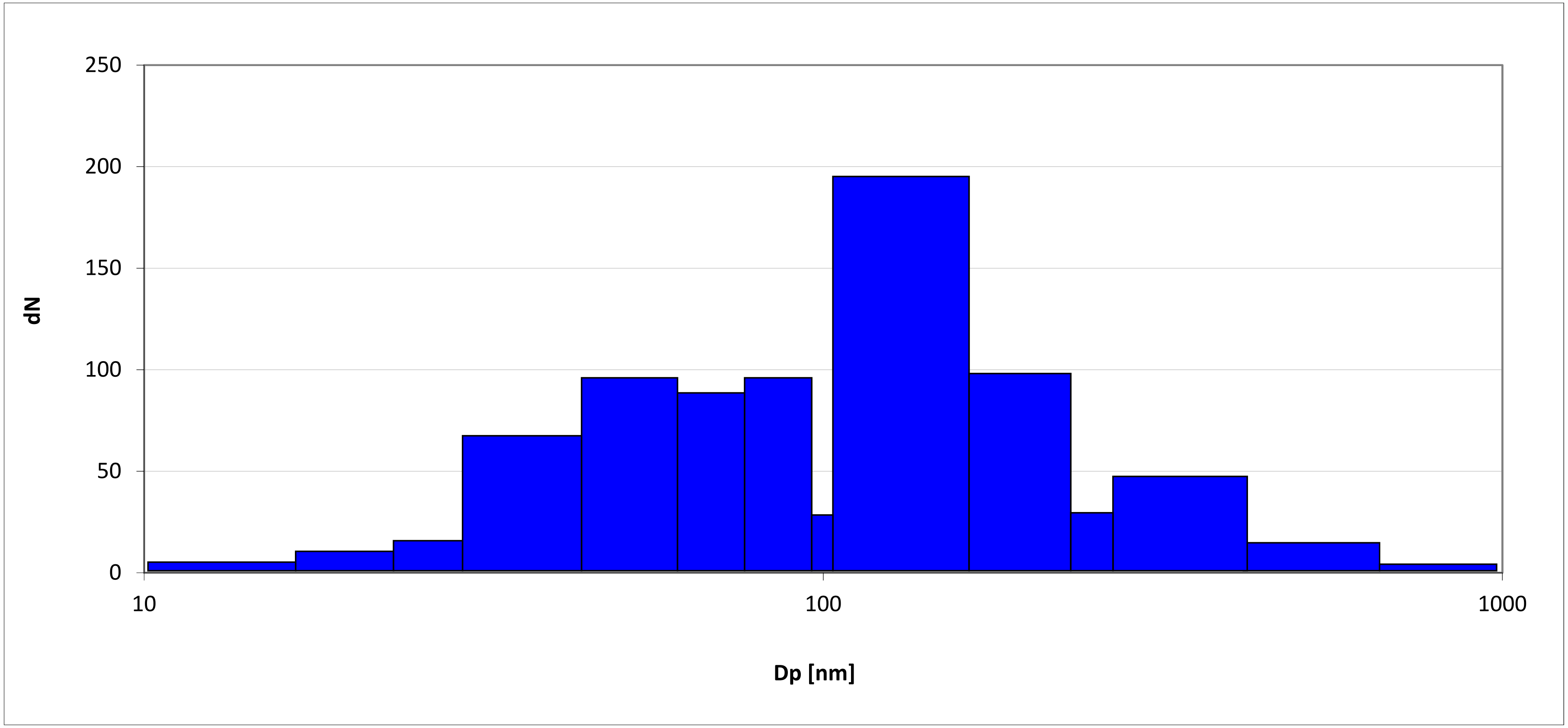
Histogram: Number concentrations N_i in each size interval

It is possible to plot the size distribution as number concentration over particle size. The total number concentration N is the sum of the concentrations N_i of all intervals.



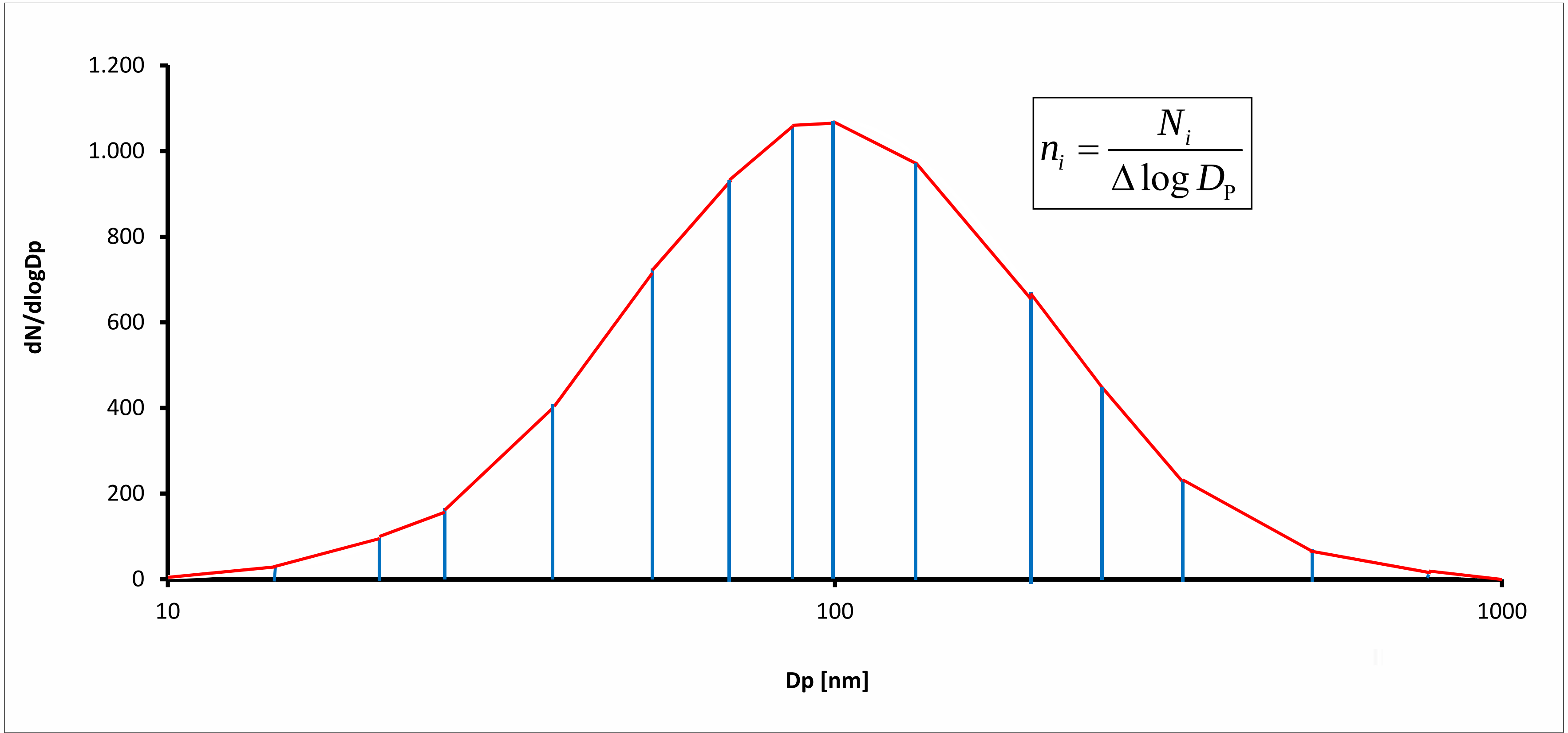
Histogram: Number concentrations N_i in each size class

A change from linear to logarithmic scale is an advantage, if the size range covers more than one order of magnitude (normal case for the atmospheric aerosol)



Number concentration N_i in each size class normalized by the logarithmic width of the size interval

N_i is here normalized by the logarithmic width of the size interval.



$$n_i = \frac{N_i}{\Delta \log D_P}$$

Lognormal Size Distribution

Log-Normal-Distribution

- The height of humans is normally distributed. Only few people exceed the height of 2.30 m.
- The annually income of people is log-normally distributed. There are people earning more than 1000-times more than normal workers.

Aerosol particle size distributions

- An aerosol particle size distribution can be often described as a log-normal distribution.
- Log-normal-distributions are used to plot particle size distributions, e.g. in the size range from 1-1000 nm.
- Normal- und log-normal-distributions are probability functions.
- The area under a (log)-normal distribution function is unity.
- The log-normal must be multiplied with the total number concentration to describe the particle number distribution.

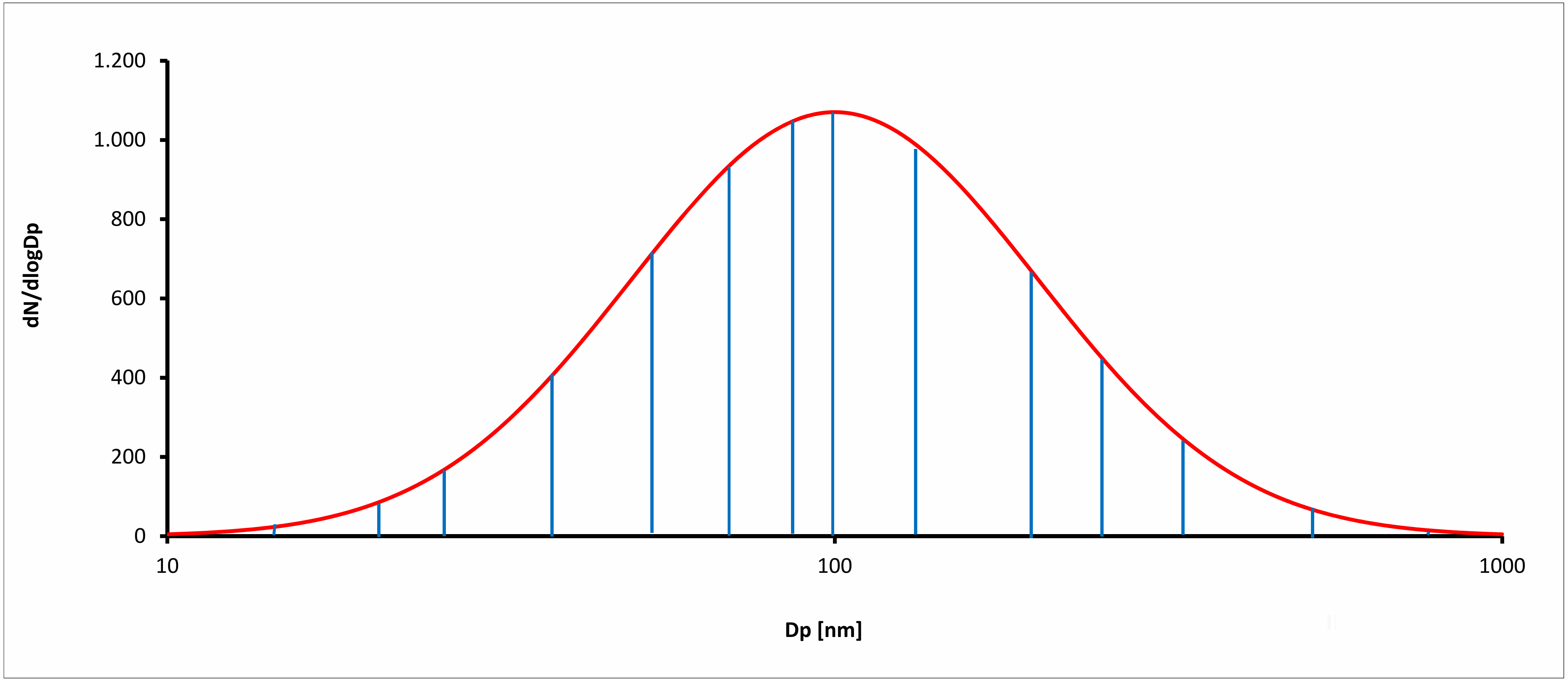
The log-normal-distribution is described by the geometric mean diameter and standard deviation.

$$\frac{\mathrm{d}f}{\mathrm{d}\log D_\mathrm{P}} = \frac{N}{\sqrt{2\pi \cdot \log \sigma_g}} \exp\left(-\frac{\left(\log D_\mathrm{P} - \log \overline{D_{\mathrm{P}_g}}\right)^2}{2\left(\log \sigma_g\right)^2} \right)$$

$$\log \overline{D_{\mathrm{P}_g}} = \frac{\sum N_i \cdot \log D_{\mathrm{P}_i}}{N}$$

$$\log \sigma_g = \left(\frac{\sum N_i \left(\log D_\mathrm{P} - \log \overline{D_{\mathrm{P}_g}}\right)^2}{N-1} \right)^{1/2}$$

Particle number size distribution: Log-Normal function



Recommendations

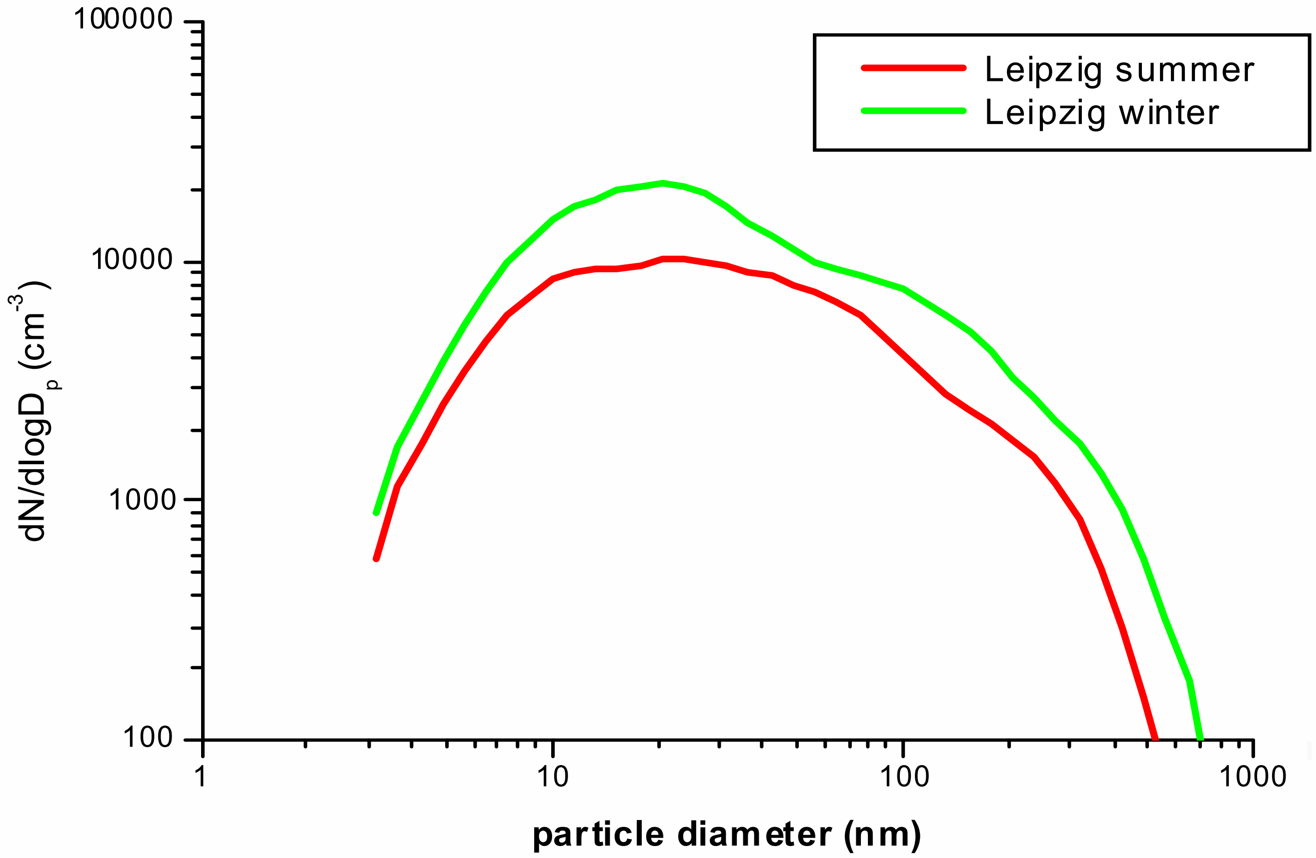
Recommendations

- If the size range (x-axis) covers 1 order of magnitude, use a linear scale.
- If the size range covers 2 or more orders of magnitude, use the logarithmic scale.
- To display a concentration (y-axis), use always the normalized style (e.g. $\Delta N/\Delta D_P$).
- If the concentration range of interest covers 1 order of magnitude, use a linear scale.
- If the concentration range of interest covers 2 or more orders of magnitude, use the logarithmic scale.

Examples of Atmospheric Particle Number Size Distributions

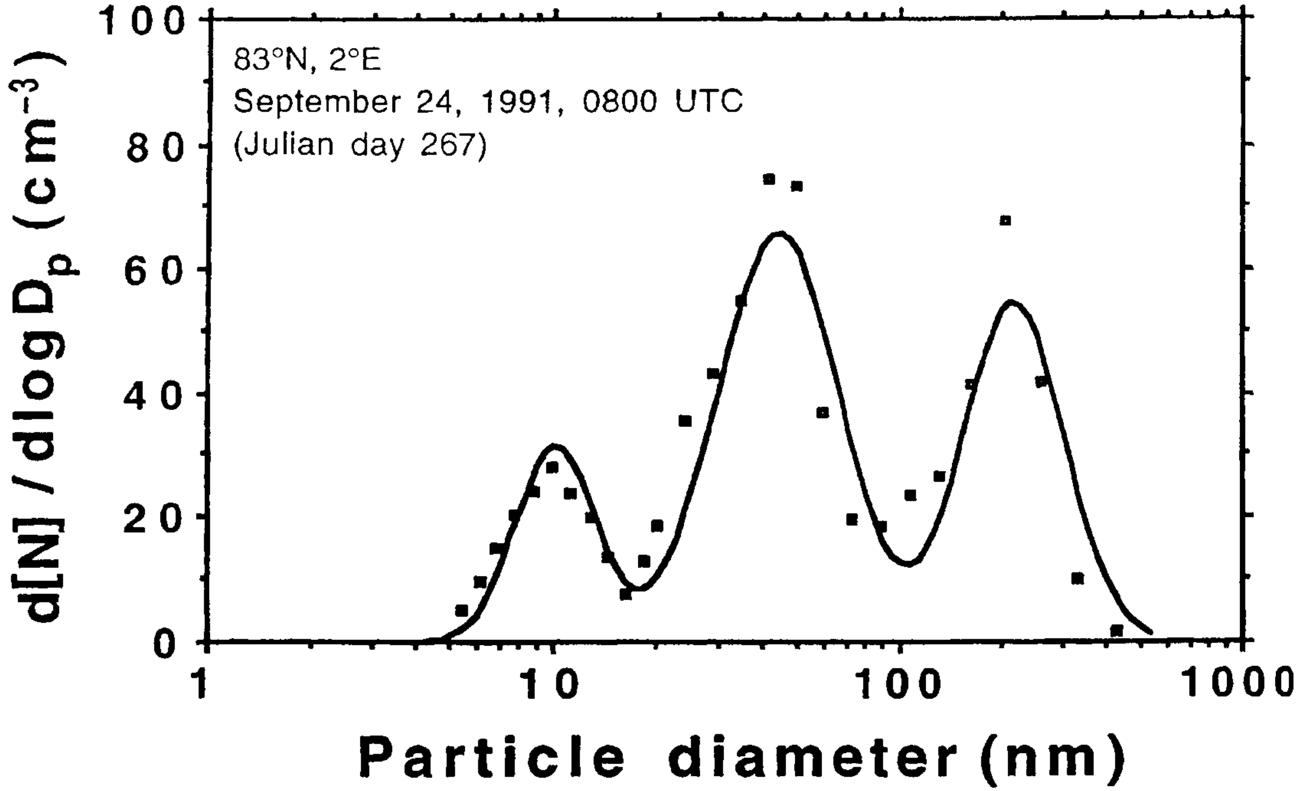
Leipzig

- Mean particle number size distribution measured in Leipzig, Germany, for winter and summer
- Double logarithmic axes



Arctic

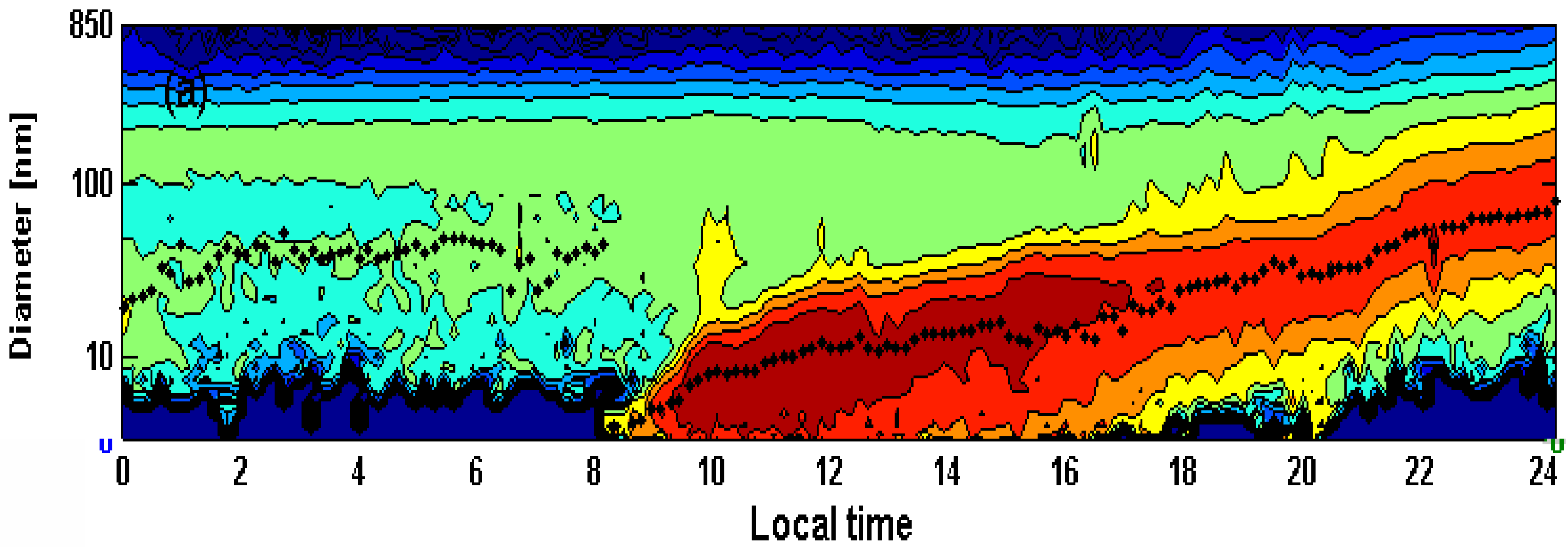
- Particle number size distribution with three log-normal modes measured in the Arctic
- x-axis logarithmic, y-axis linear



Covert, D. S. et al. (1995) Tellus **48B**, 197-212.

North China Plain

- Contour plot of the diurnal development of the particle number size distribution after a new particle formation event



Shen, X. J. et al. (2011) ACP **11**, 1565-1580