Part 1
Radiative Transfer

Mini-curso Lidar
Ceilometer

Henrique M. J. Barbosa
Instituto de Física – USP
hbarbosa@if.usp.br
http://www.fap.if.usp.br/~hbarbosa

Interaction of Light with matter

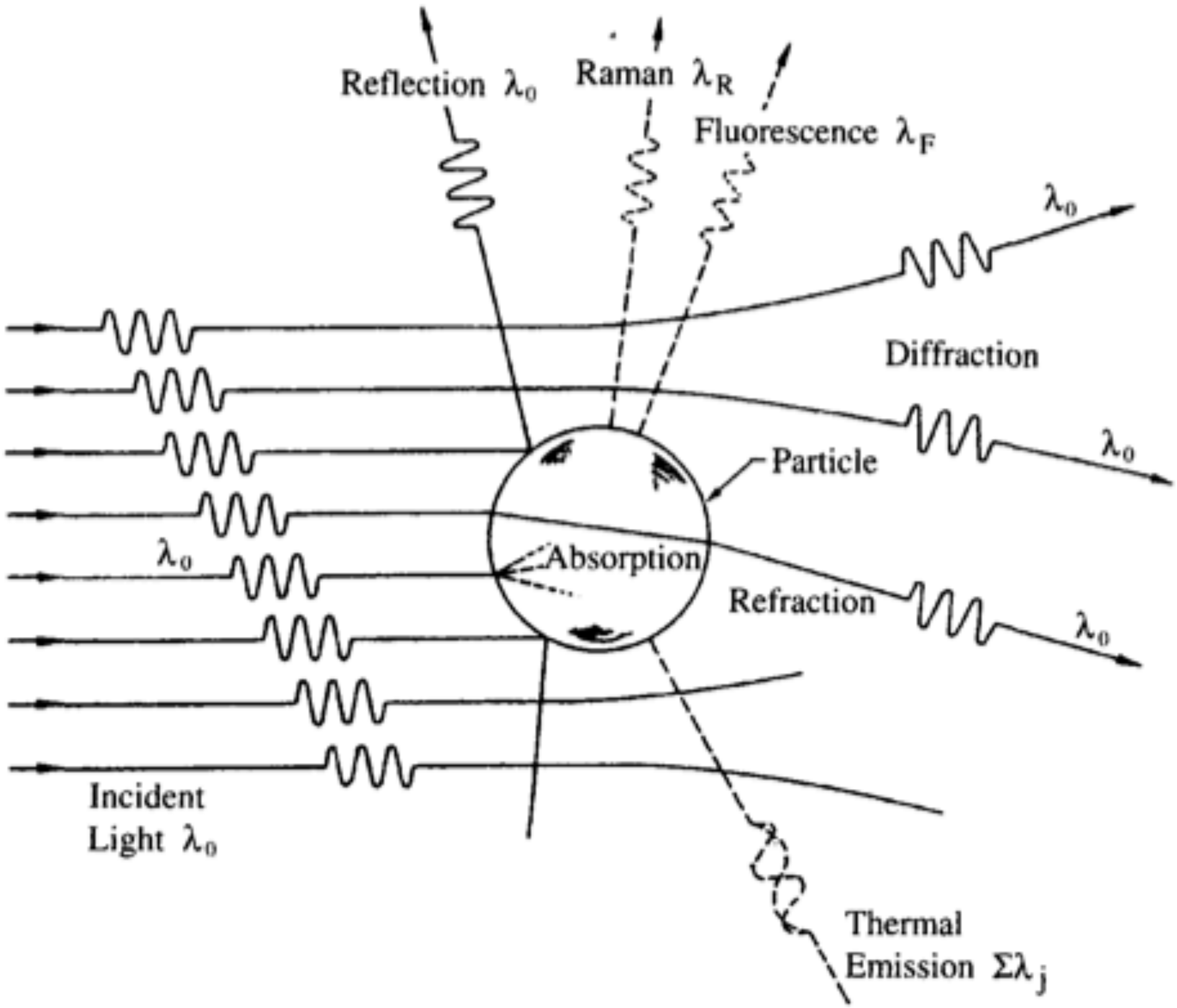


FIGURE 15.1 Mechanisms of interaction between incident radiation and a particle.

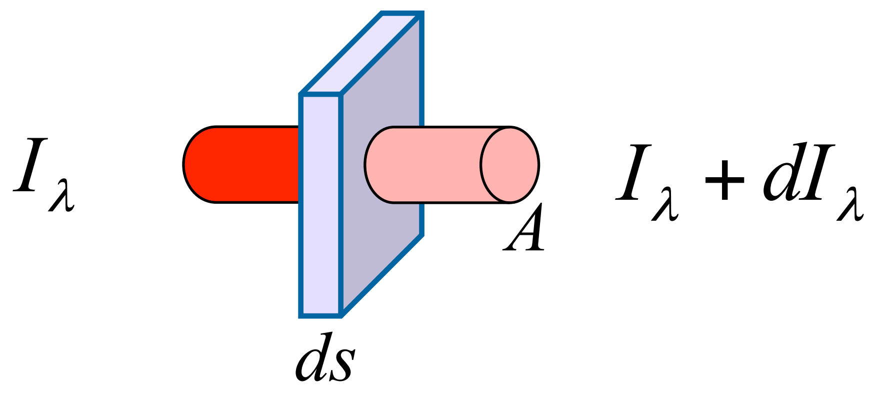
Seinfeld & Pandis, chapter 15

Radiation in the Atmosphere

- Extinction:
  - It is a process that reduces the radiance. Can be due to **absorption** or to **scattering**
    - **Absorption: transforms EM energy in something else**
    - **Scattering: changes direction of propagation**
- Emission:
  - It is a process that increases the radiance.
    - All bodies with **T>0 K** emits radiation
    - There can be scattered radiation in the beam direction

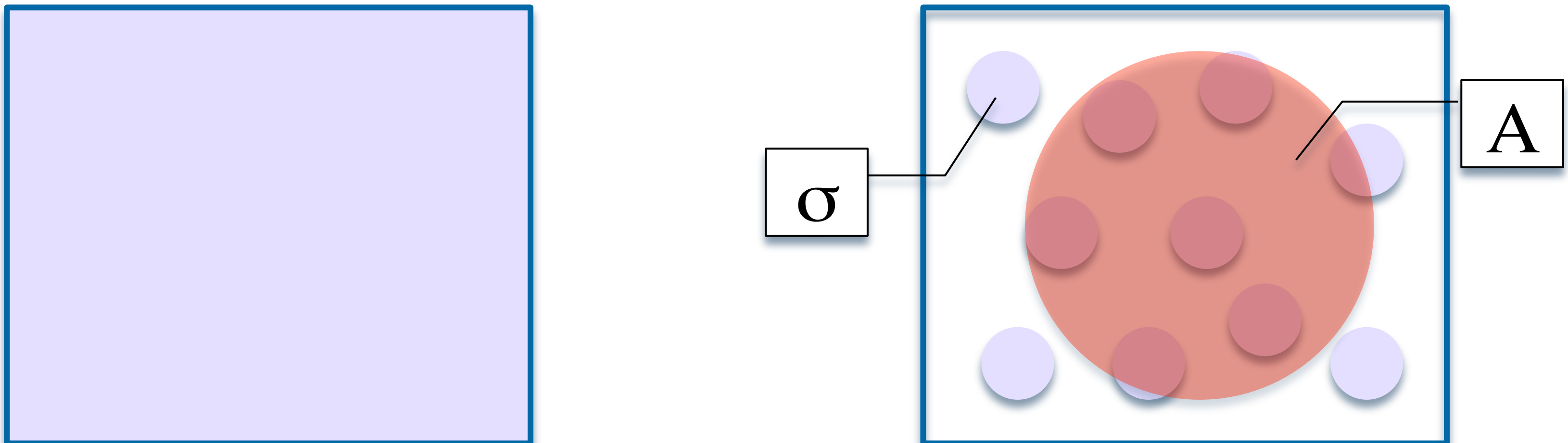
Light extinction

- The extinction process is proportional to the radiance and to the amount of matter



- If ds is small enough, there is no overlap between scatters (single layer limit)

Light extinction



$ds \rightarrow \infty$ $\qquad\qquad$ $ds \rightarrow 0$

- Therefore, the fraction of extinguished photons is

$$\frac{dI_\lambda}{I_\lambda} = -\frac{\sigma \cdot NAds}{A}$$

σ = cross section
N = # / volume
N A ds = #

Light extinction

- Therefore, in terms of <u>extinction cross section</u> [m^2], **σ**:

$$dI_\lambda = -\sigma N I_\lambda ds$$

- Or in terms of <u>volume-extinction coefficient</u> [1/m], **α**:

$$dI_\lambda = -\alpha I_\lambda ds$$

- Or in terms of <u>mass extinction coefficient</u> [m^2/kg]:

$$dI_\lambda = -(\alpha / \rho) I_\lambda d\chi \qquad d\chi = \rho \cdot ds$$

Mass thickness

- Or in terms of <u>molar extinction</u> [m^2/mol], **ε**:

$$dI_\lambda = -\varepsilon c I_\lambda ds$$

c = Molar concentration

Extinction along a path

- Solving this differential equation, we find:

$$I_\lambda(s) = I_\lambda(s_0)\exp\left[-\underbrace{\int_{s_0}^{s}\alpha(\lambda,s')\,ds'}\right]$$

- … and if there is different types of particles:

$$I_\lambda(s) = I_\lambda(s_0)\exp\left[-\underbrace{\sum_i\int_{s_0}^{s}\alpha_i(\lambda,s')\,ds'}_{\tau(s_0,s)}\right]$$

Beer-Bouguer-Lambert Law

$$I = I_0 \mathrm{e}^{-\tau}$$

- **Wikipedia** - *This law was discovered by Pierre Bouguer before 1729 and it is often (mis)attributed to Johann Heinrich Lambert, who cited Bouguer's "Essai d'Optique sur la Gradation de la Lumiere" (Claude Jombert, Paris, 1729), and even quoted from it, in his "Photometria" in 1760. Much later, August Beer extended the exponential absorption law in 1852 to include the concentration of solutions in the absorption coefficient.*

$$dI_\lambda / I_\lambda \propto ds$$

Bougher

$$dI_\lambda / I_\lambda \propto c$$

Beer

Points to remember #1

- We will see that we do not have to solve the radiative transfer equation in the case our a LIDAR system…
- What is important from this review is:

$$I(\lambda) = I_0(\lambda)\mathrm{e}^{-\tau_{total}}$$

$$\tau_{total}(\lambda) = \sum_{k=species} \int_{s_0}^{s} \alpha_{total}^{k}(\lambda, s')\, ds'$$

$$\alpha = N\sigma \Rightarrow \sigma_{total} = \sigma_{abs} + \sigma_{scat}$$

Extinction efficiency

- The cross-section has units of area (that "*shadows*" the light), but this "*area*" can be much larger or smaller then the real area, A_e, of the extinguisher.

$$\sigma_{ext} = \sigma_{abs} + \sigma_{scat} \qquad [L^2]$$

- We can define dimensionless ***scattering*** and ***absorbing efficiencies*** by making:

$$Q_{abs} = \frac{\sigma_{abs}}{A_e} \qquad Q_{scat} = \frac{\sigma_{scat}}{A_e}$$

Single scattering albedo

- The ratio of $\mathbf{Q_{scat}}$ and $\mathbf{Q_{ext}}$ is called the single-scattering albedo, ω, and represents the fraction of light extinction due to the scattering processes:

$$\omega = \frac{Q_{scat}}{Q_{ext}} = \frac{\sigma_{scat}}{\sigma_{ext}} = \frac{\sigma_{scat}}{\sigma_{scat} + \sigma_{abs}}$$

- Hence, $\mathbf{1-}\omega$, is the fraction that is absorbed.

Angstrom exponent

To obtain the extinction coefficient at the transmitted wavelength we have to introduce the Ångström exponent $\mathring{a}(R)$, which describes the wavelength dependence of the particle extinction coefficient,

$$\frac{\alpha_{\text{aer}}(\lambda_0)}{\alpha_{\text{aer}}(\lambda_{\text{Ra}})} = \left(\frac{\lambda_{\text{Ra}}}{\lambda_0}\right)^{\mathring{a}(R)}, \tag{4.18}$$

Table 4.1. Properties of aerosol types [1][a]

Aerosol type	N (cm^{-3})	r_{eff} (μm)	ssa (0.55 μm)	g (0.55 μm)	$\mathring{a}$ (0.35–0.55 μm)	$\mathring{a}$ (0.55–0.8 μm)
Cont. clean	2600	0.247	0.972	0.709	1.10	1.42
Cont. average	15,300	0.204	0.925	0.703	1.11	1.42
Cont. polluted	50,000	0.150	0.892	0.698	1.13	1.45
Urban	158,000	0.139	0.817	0.689	1.14	0.43
Desert	2300	1.488	0.888	0.729	0.20	0.17
Marit. clean	1520	0.445	0.997	0.772	0.12	0.08
Marit. polluted	9000	0.252	0.975	0.756	0.41	0.35
Marit. tropical	600	0.479	0.998	0.774	0.07	0.04
Arctic	6600	0.120	0.887	0.721	0.85	0.89
Antarctic	43	0.260	1.000	0.784	0.34	0.73
Stratosphere (12–35 km)	3	0.243	1.000	0.784	0.74	1.14

Weitkamp, chap 4

Light scattering

- Absorption and elastic scattering of light by a spherical object is a classical problem in physics.
- The key parameters that govern scattering and absorption of light by an sphere are

1. The wavelength λ
2. The diameter of the sphere D
3. Index of refraction of the sphere

$$x = \pi D / \lambda$$

$$\tilde{n} = n + i\kappa$$

- The mathematical formalism used to solve this problem is called **Mie Theory**.

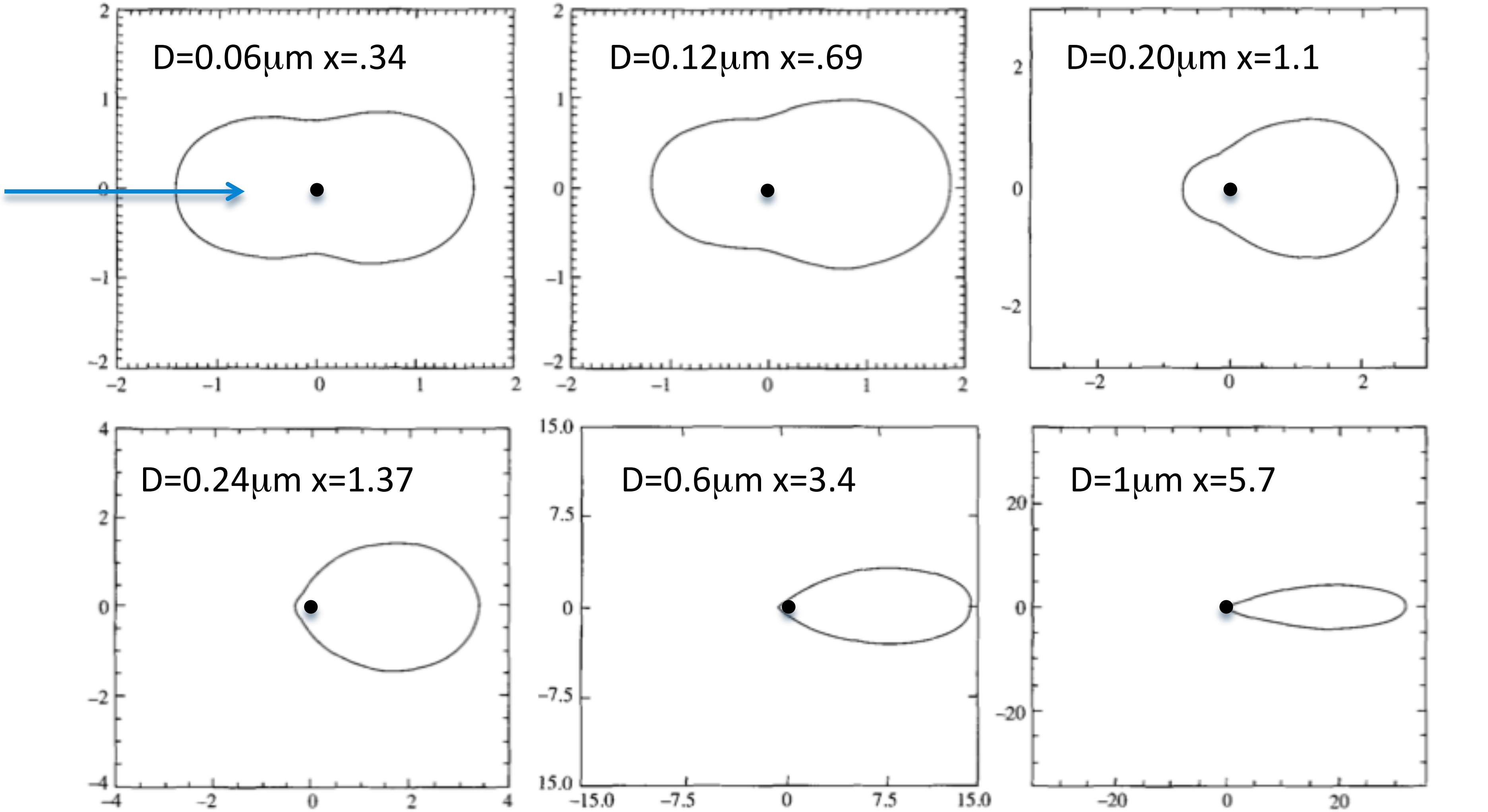
Mie Theory

Mie theory is the basis of a computational procedure to calculate the scattering and absorption of light by any sphere as a function of wavelength.

- Limiting cases:
  - **$\pi D/\lambda$ <<** 1 Rayleigh scattering
    - $Q_{scat} \sim \lambda^{-4}$ and $Q_{abs} \sim \lambda^{-1}$
  - **$\pi D/\lambda$ ~** 1 Mie scattering
    - Q_{scat} and Q_{abs} vary a lot with **x** and **ñ**
  - **$\pi D/\lambda$ >>** 1 Geometric optics
    - Reflection, refraction and diffraction

Phase function

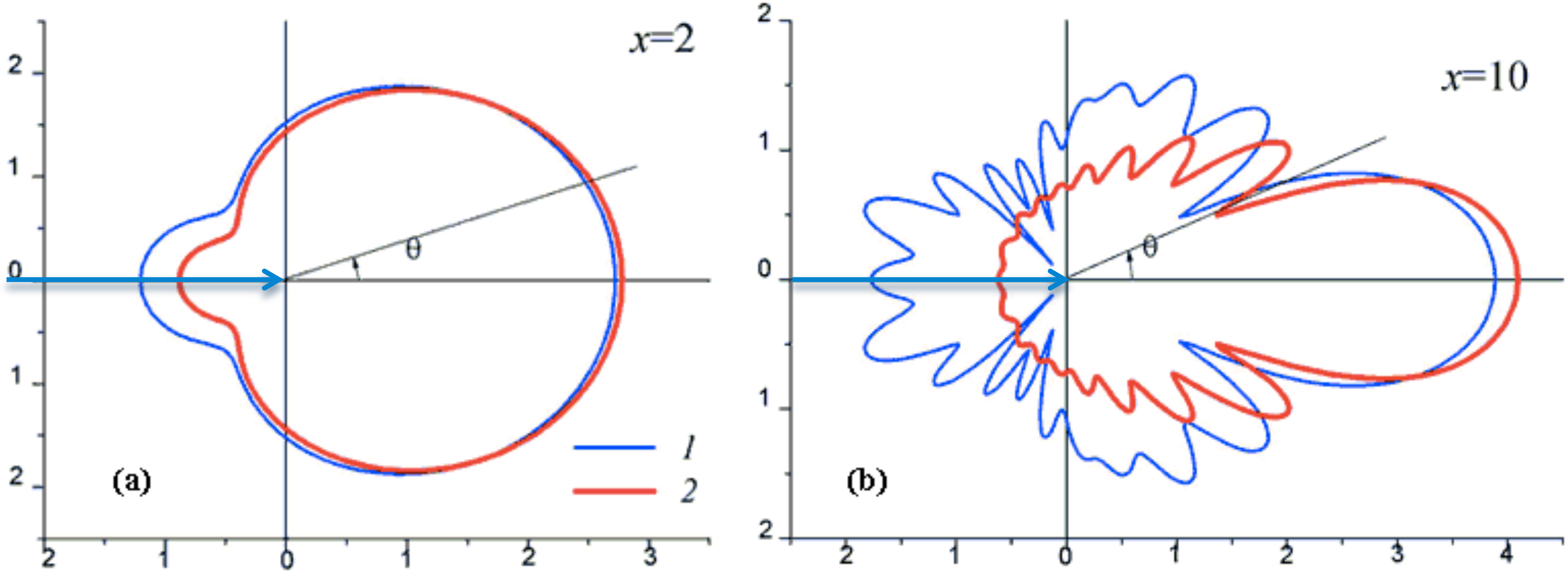
Example
$(NH_4)_2SO_4$ RH=80% λ=550nm



Seinfeld & Pandis, cap. 15

Phase function

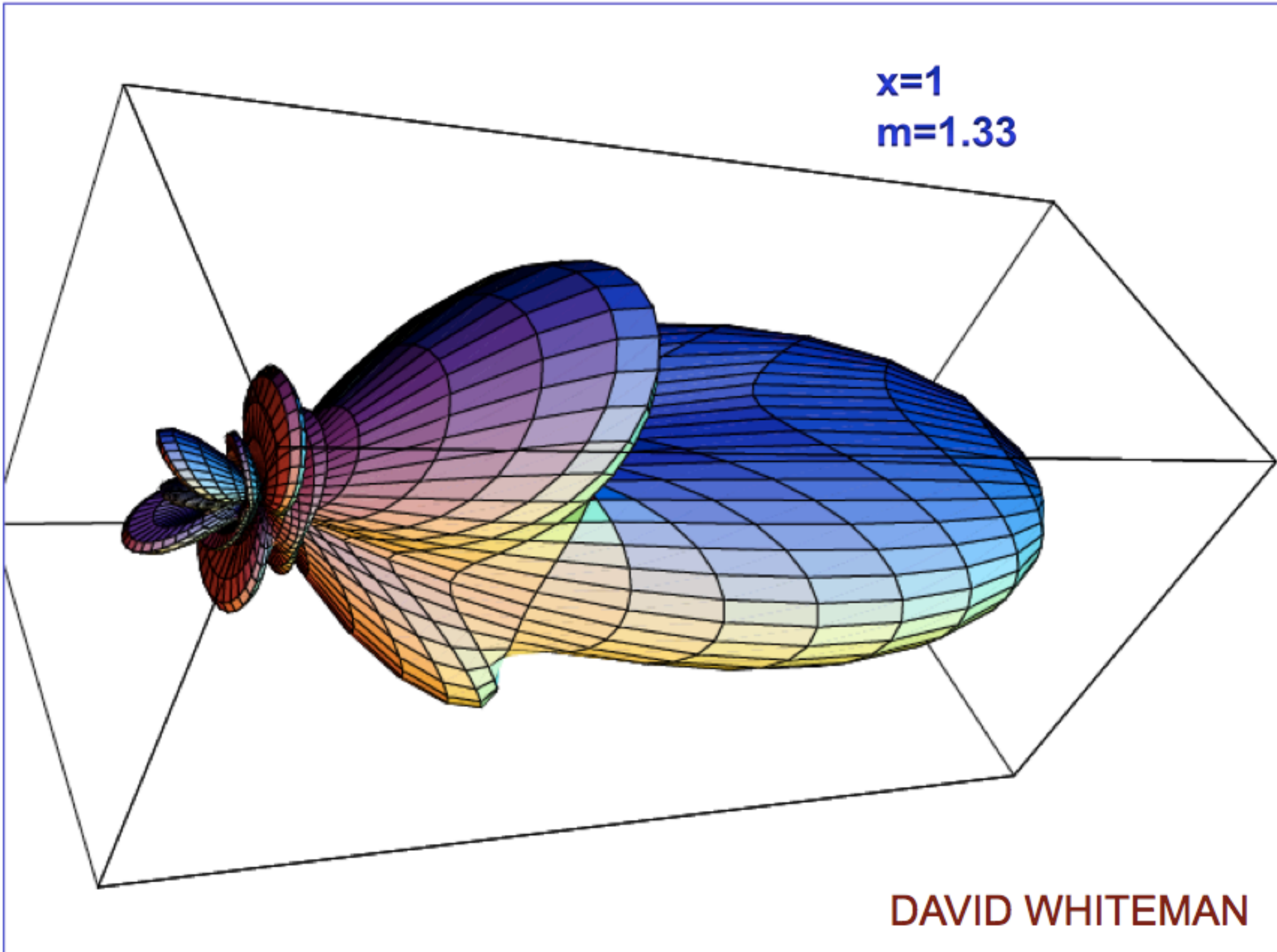
The angular distribution of light intensity scattered by a particle at a given wavelength is called the *phase function.*



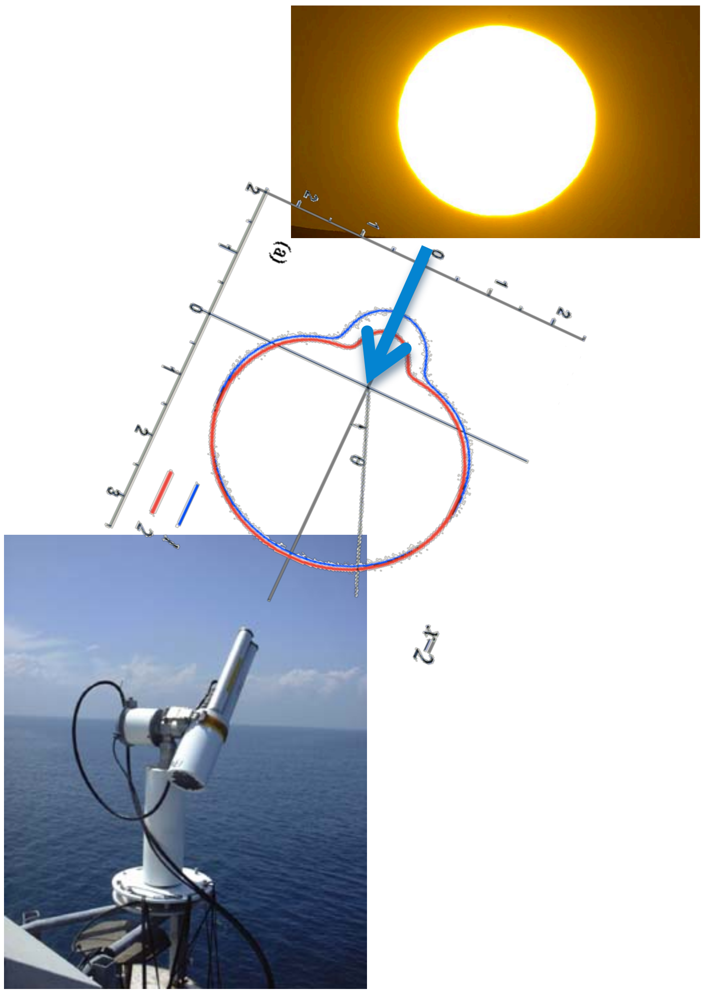
ñ = 1.5 + i*0.005

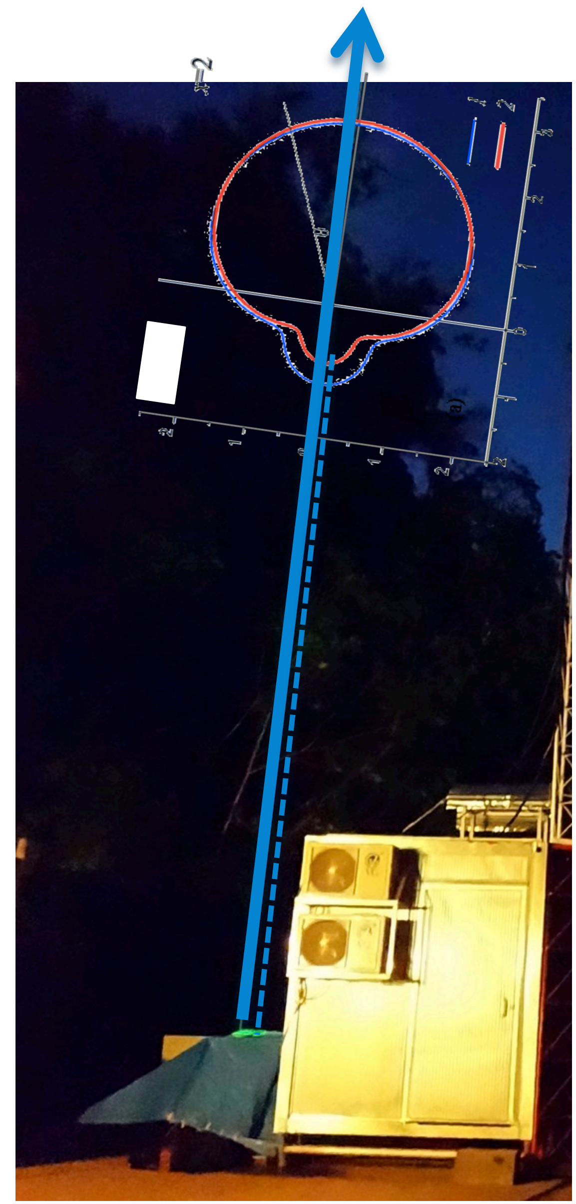
ñ = 1.5 + i*0.2

http://www.thermopedia.com/content/144/


x=1
m=1.33
DAVID WHITEMAN

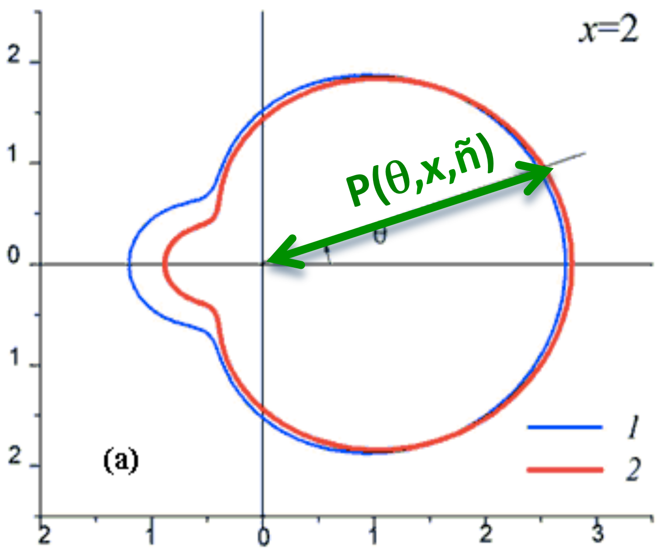
Lidar x Sunphotometer





Phase function

It is the scattered intensity at a particular angle θ normalized by the total scattered intensity considering all angles.



$$\int_0^{2\pi}\int_0^{\pi} P(\theta,x,\tilde{n})\sin\theta\,d\theta\,d\varphi = 4\pi$$

The **angular volume-scattering coefficient** [m^{-1} sr^{-1}], β, is

$$\beta(\theta,x,\tilde{n},\lambda) = \alpha_{scat}(\lambda)\frac{P(\theta,x,\tilde{n})}{4\pi}$$

Asymmetry parameter

- The asymmetry parameter **g** is defined as the intensity-weighted average of the cosine of the scattering angle:

$$g = \frac{1}{2}\frac{\int_0^{\pi} \cos\theta \; F(\theta) \sin\theta \, d\theta}{\int_0^{\pi} F(\theta) \sin\theta \, d\theta}$$

$$= \frac{1}{2}\int_0^{\pi} \cos\theta \; P(\theta) \sin\theta \, d\theta$$

- The factor of ½ ensures that **g = 1** for light scattered totally at **θ=0°** (forward) and **g = -1** for light scattered completely at **θ=180°** (backward).

Points to remember #2

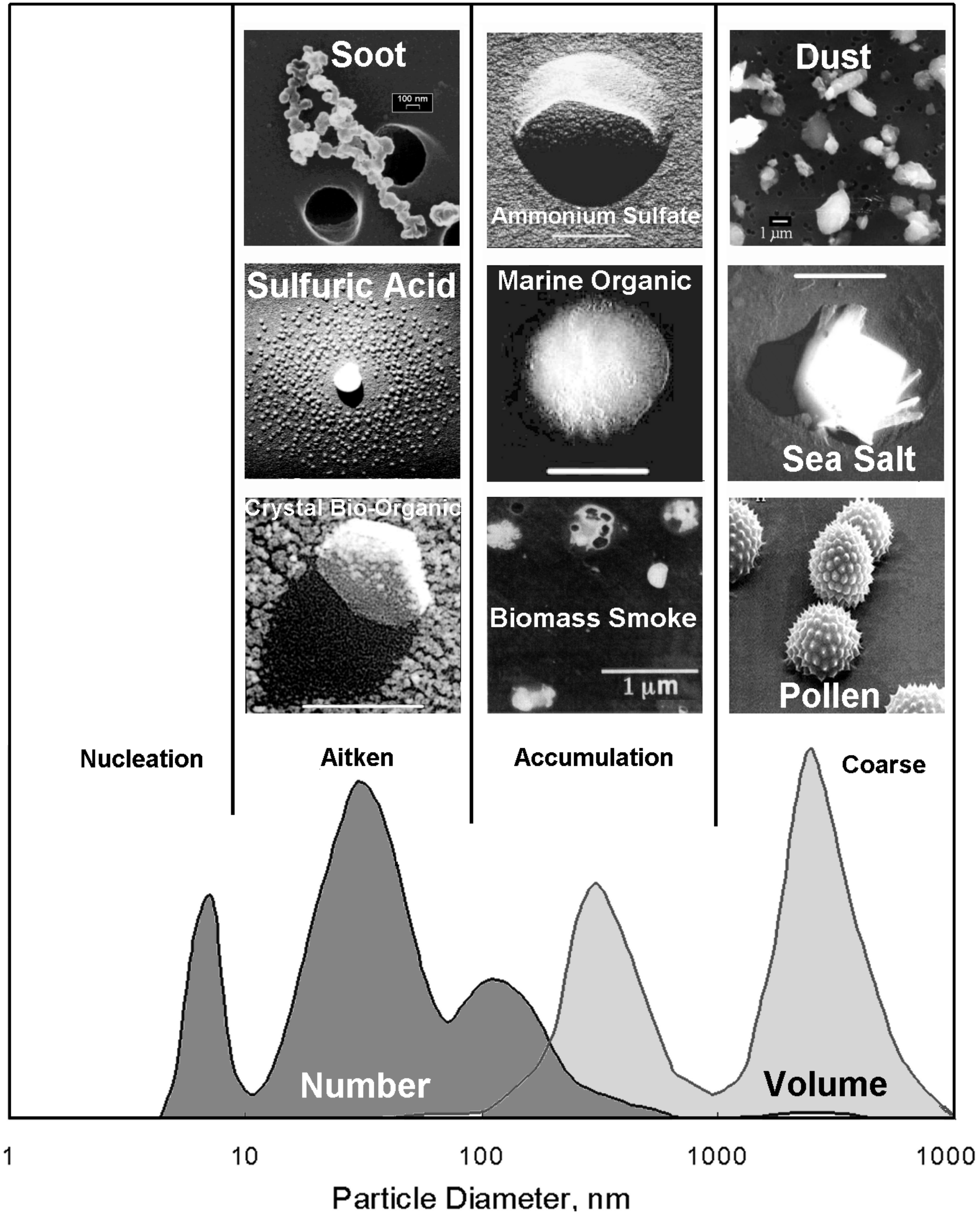
- Aerosol and molecule interaction with the radiation depends on:
  - Size

$$x = \pi D / \lambda$$

  - Shape

  - Surface properties

$$\tilde{n} = n + i\kappa$$



Points to remember #3

- The atmosphere has
  - Molecules (~ 0.1 to 0.5 nm)
  - Aerosol particles (10nm to 1μm)
- Solar radiation ~ 0.1 to 4μm, hence 2 types of scattering
  - **πD/λ <<** 1 Rayleigh scattering - **Molecules**
  - **πD/λ ~** 1 Mie scattering - **Aerosols**
- Total extinction is given by

$$\alpha_{ext} = \alpha_{abs} + \alpha_{scat} \qquad \begin{cases} \alpha_{abs} = \alpha_{abs,g} + \alpha_{abs,p} \\ \alpha_{scat} = \alpha_{scat,g} + \alpha_{scat,p} \end{cases}$$