Part 3
Lidar Equation

Mini-curso Lidar
Ceilometer

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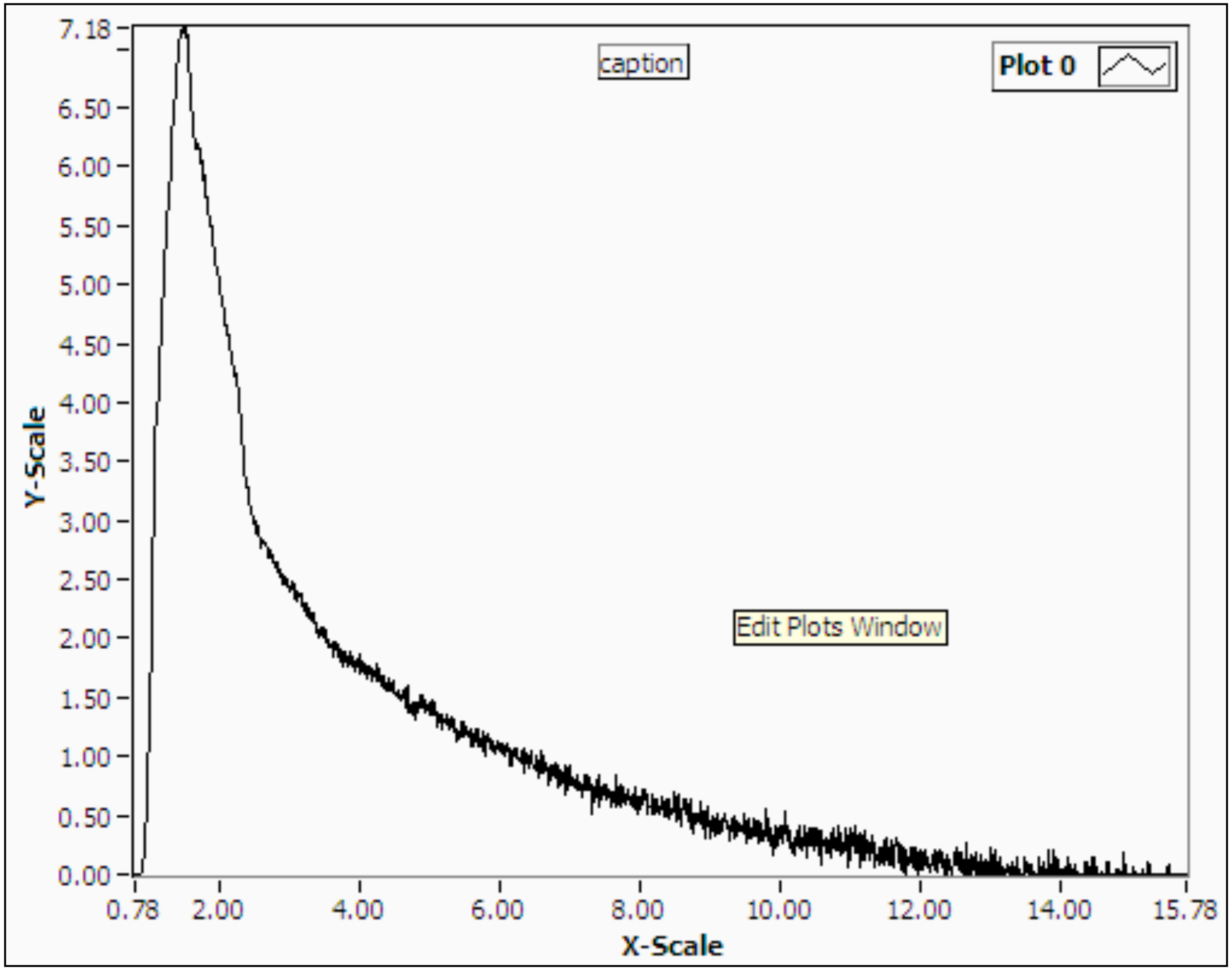
How can we describe this signal?

How does the signal vary with time (or height) ?



The simplest lidar equation

$$P(r) = K \cdot G(r) \cdot B(r) \cdot T(r)$$

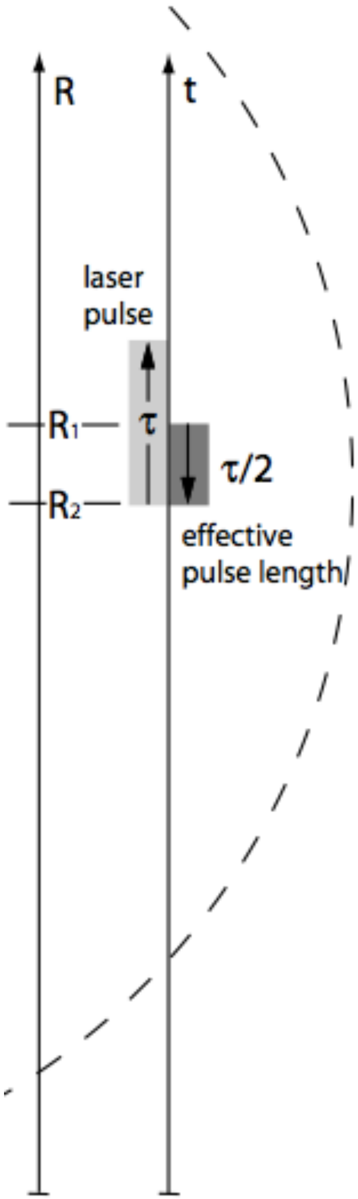
1. K = System performance
2. G(r) = Change of geometry with range r
3. B(r) = Fraction of light scattered towards the telescope
4. T(r) = Atmospheric transmission

(1) System performance

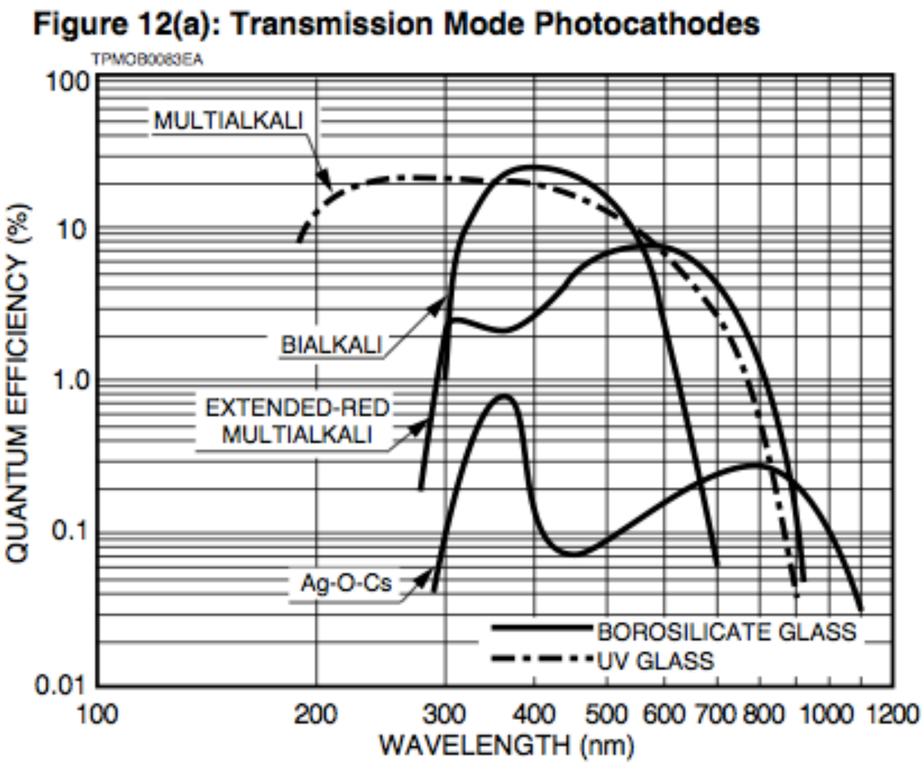
- Number of photons emitted $= P_0$

- Detection efficiency $= \eta(\lambda)$

- Effective pulse length $= \dfrac{c\tau}{2}$



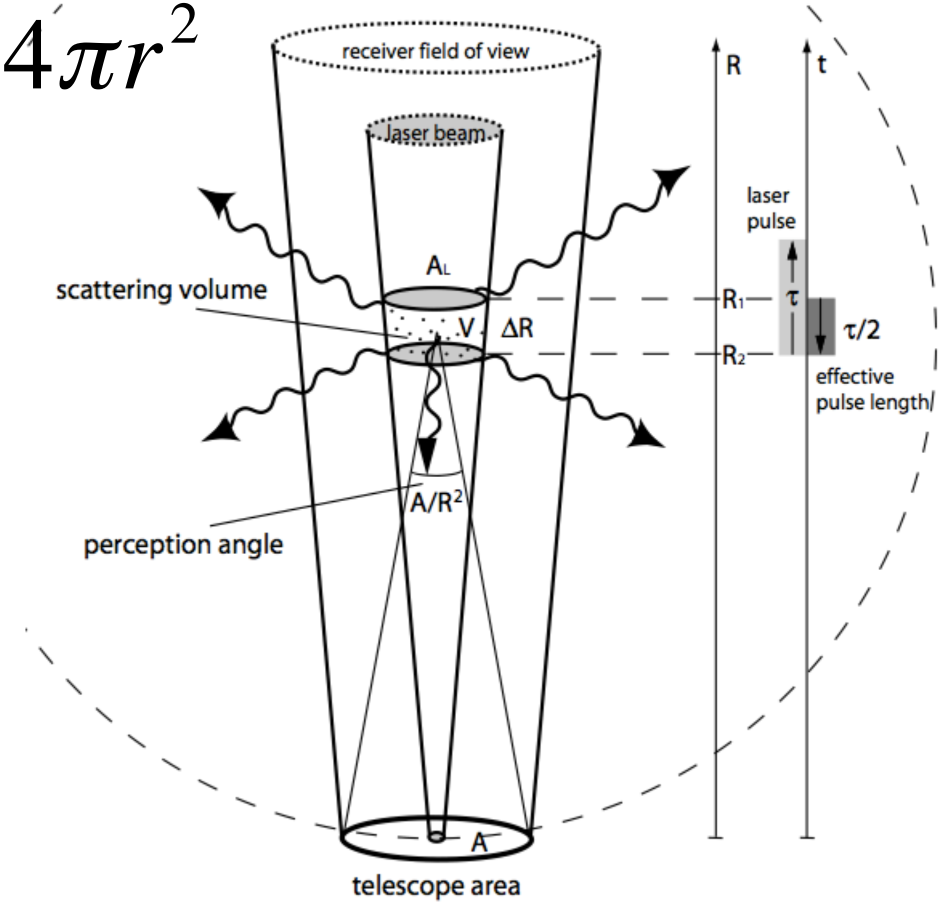
(1) System performance



And of course: windows, mirrors, filters, …

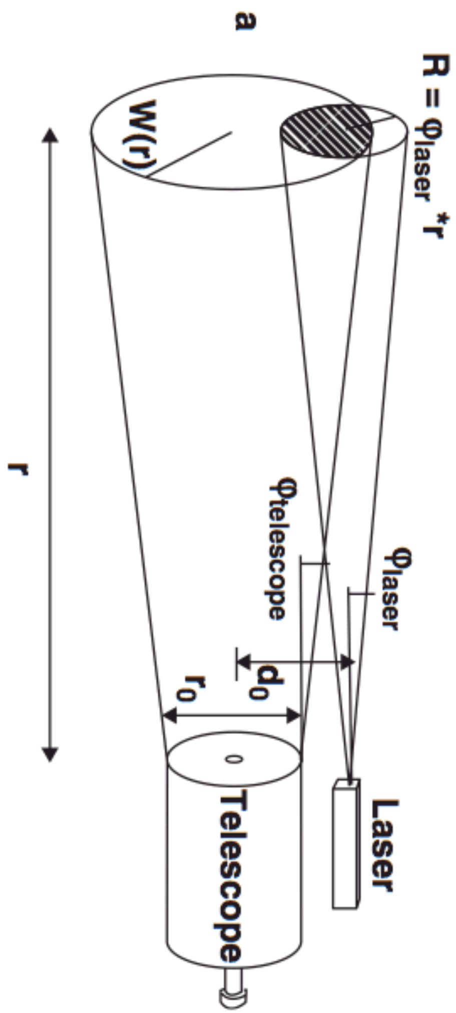
(2) Change of Geometry

- Solid angle subtended $= \dfrac{A}{4\pi r^2}$
- Overlap factor $= O(r)$



Weitkamp, chap 1

(2) Change of Geometry



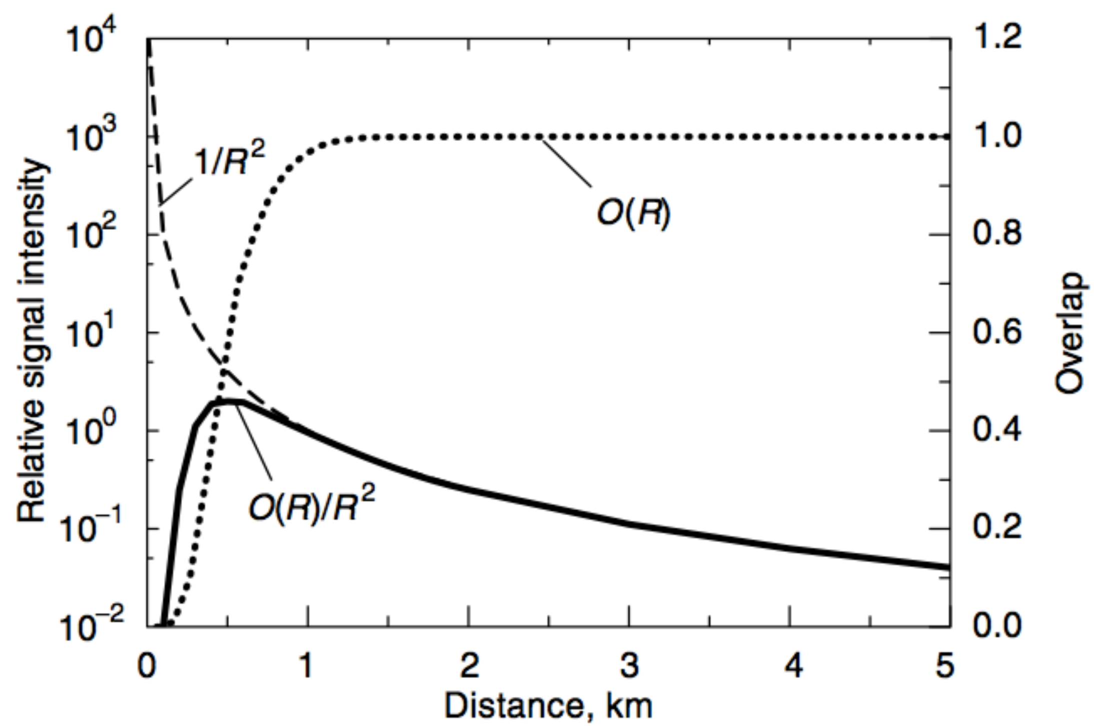


Fig. 1.3. Influence of the overlap function on the signal dynamics.

Kovalev, "Elastic Lidar", chap. 3

Weitkamp, chap 1

(3) Back-scatter coefficient

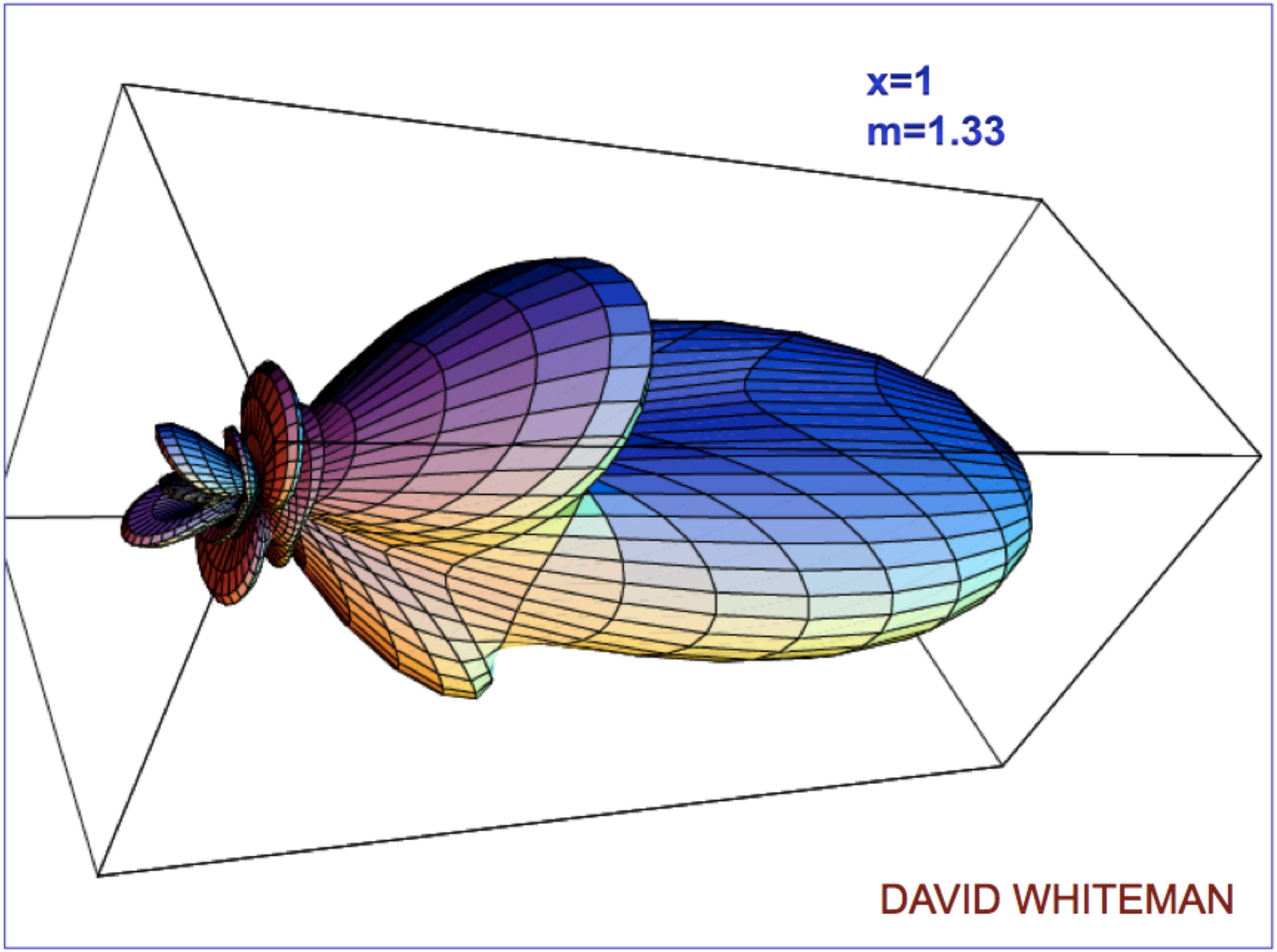
- For any angle θ, we had:

$$\beta(\theta, x(r), \tilde{n}(r), \lambda) = \alpha_{scat}(\lambda)\frac{P(\theta, x(r), \tilde{n}(r))}{4\pi}$$

- Telescope is small, **r>>1** and **θ~π** (backscatter), and for isotropic scattering (**P=1**), the total scattering is then:

$$4\pi\beta(\pi, r, \lambda) = N(r)\sigma_{scat}(\lambda)$$

(3) Back-scatter coefficient



(4) Transmission Term

- As the laser pulse travels two times the distance from the Lidar to range **r**, then the transmission term is simply:

$$T(r,\lambda) = \exp\left[-2\int_0^r \alpha_{ext}(r',\lambda)\,dr'\right]$$

Full lidar equation

Points to remember #4

- Putting all these terms together we find

$$P(r,\lambda) = P_0 \frac{c\tau}{2} A\eta(\lambda) \frac{O(r)}{r^2} \beta(r,\lambda) \exp\left[-2\int_0^r \alpha_{ext}(r',\lambda)\, dr'\right]$$

- And we need to remember that

$$\beta = \beta_{mol} + \beta_{par}$$

$$\alpha_{ext} = \alpha_{mol,ext} + \alpha_{par,ext}$$

1 equation and
4 unknowns

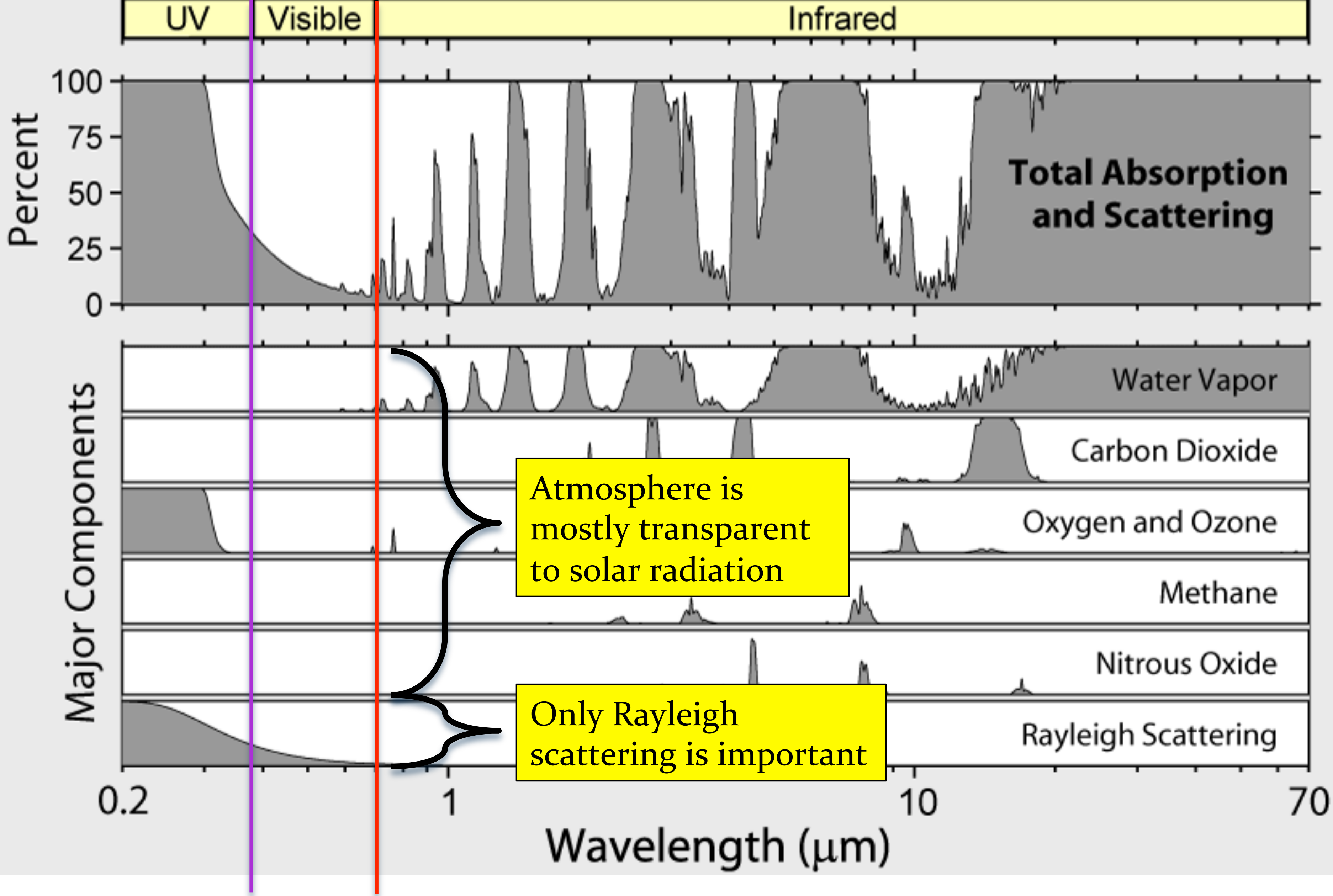
Solution to the Lidar equation

- 1 Equation and 4 unknowns
- First:
  - We know how EM waves interacts with molecules

$$\beta_{mol} \quad \alpha_{mol,ext}$$

- Second:
  - We will need to assume something about the aerosol particles

$$\beta_{par} \quad \alpha_{par,ext}$$



$$\alpha_{mol,ext} = \alpha_{mol,scat} + \alpha_{mol,abs} \approx \alpha_{mol,scat}$$

Molecular cross-section

$$\alpha = N\sigma$$

$$\sigma_{mol}(\lambda) = \frac{24\pi^3}{\lambda^4 N_{std}^2} \frac{(n_{std}^2 - 1)^2}{(n_{std}^2 + 2)^2}$$

- $n_{std}(\lambda)$ is the index of refraction of dry air
- N_{std} is the standard molecular density

Standard Atmosphere, N_{std}

- Standard atmosphere CO_2 300ppmv, 1013hPa,15°C
  - $N_{std} = 2.54743 \times 10^{19}\ \text{cm}^{-3}$

$$\alpha_{mol}(\lambda, z) = \alpha_{mol}^{std}(\lambda) \frac{N(z)}{N^{std}}$$

(Bucholtz, 1995)

Molecular signal

- The optical depth due to molecular extinction is

$$\tau_{mol}(\lambda, r) = \int_0^r \alpha(\lambda, r')\, dr' = \alpha_{mol}^{std}(\lambda) \int_0^r \frac{N(r')}{N_{std}}\, dr'$$

- Hence the molecular lidar signal is written as

$$P_{mol}(r) \propto \frac{1}{r^2} \beta_{mol}(r) \exp\left[-2\alpha_{mol}^{std} \int_0^r \frac{N(r')}{N_{std}}\, dr' \right]$$

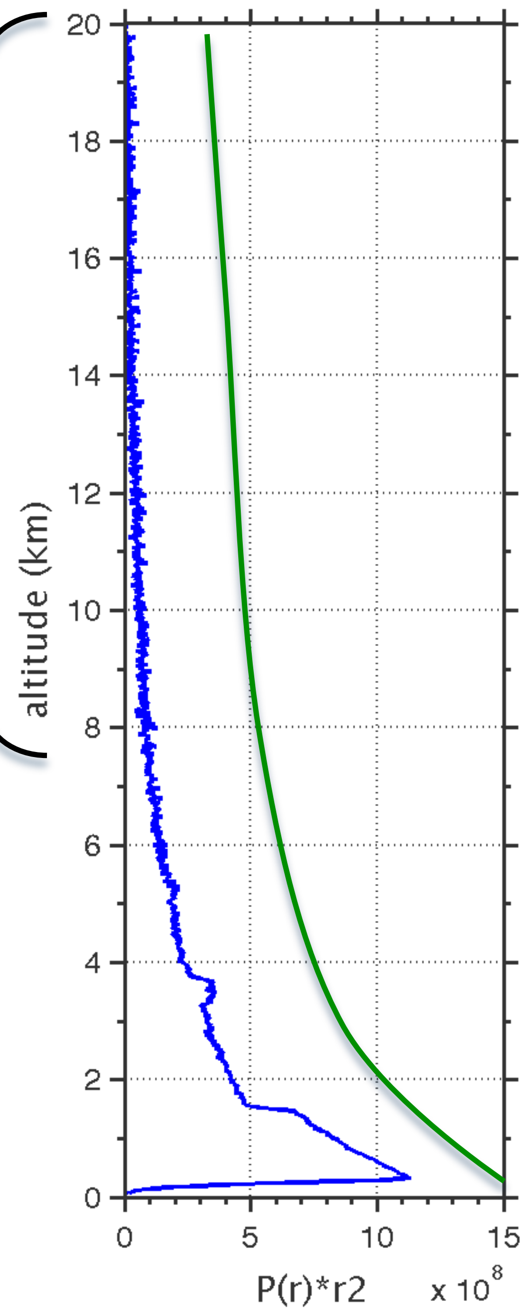
What is the proportionality constant?

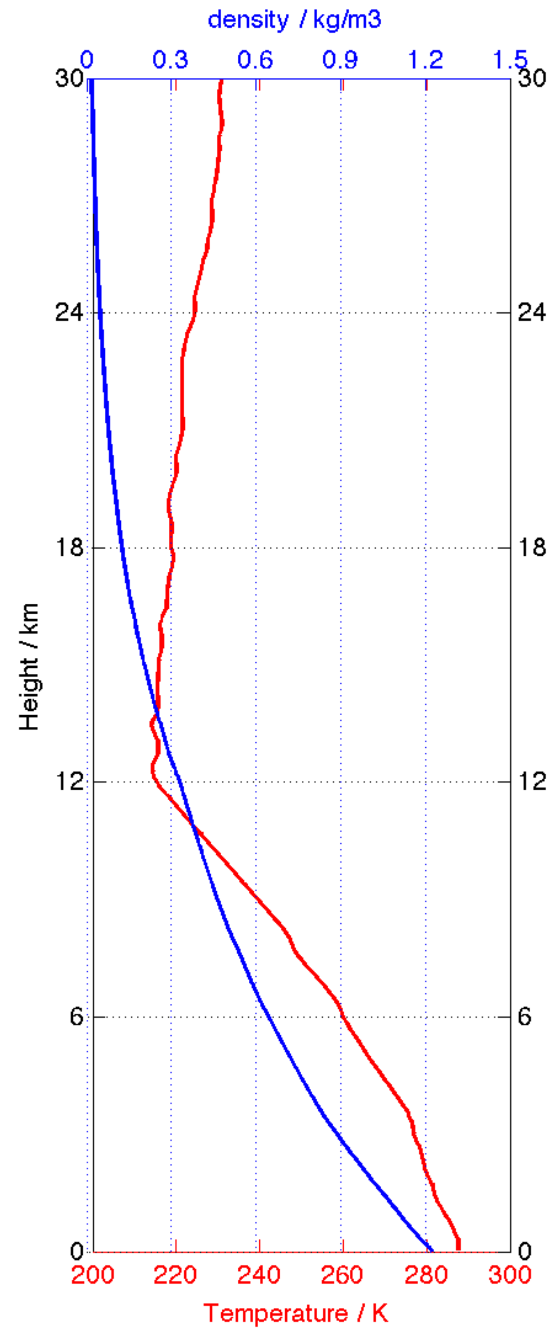
Molecular fit

This part of the signal looks like just molecular

- Fit:

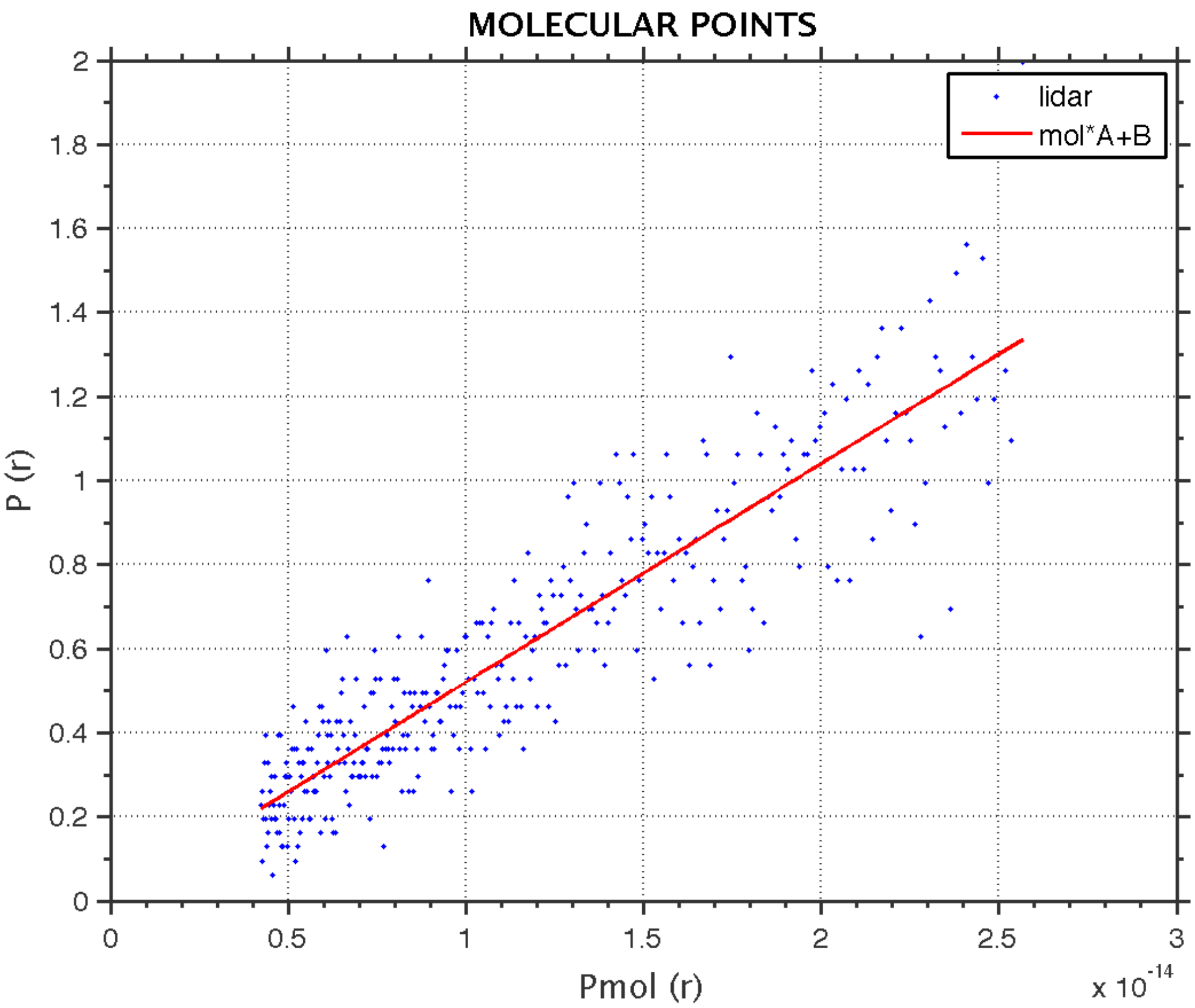
$$P(r)=A*P_{mol}(r)+BG$$

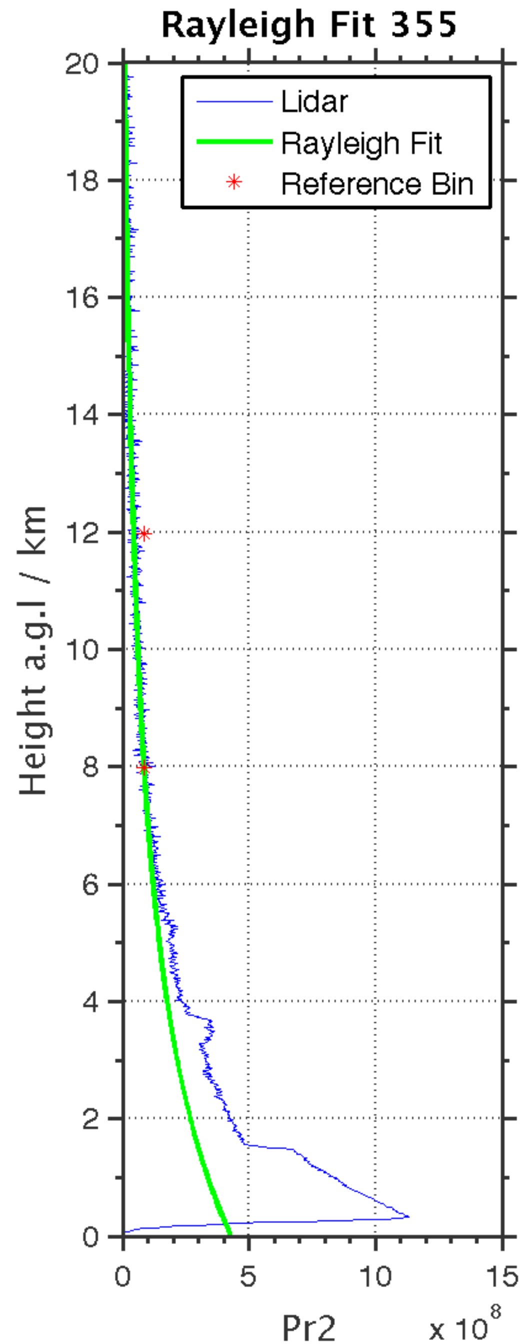




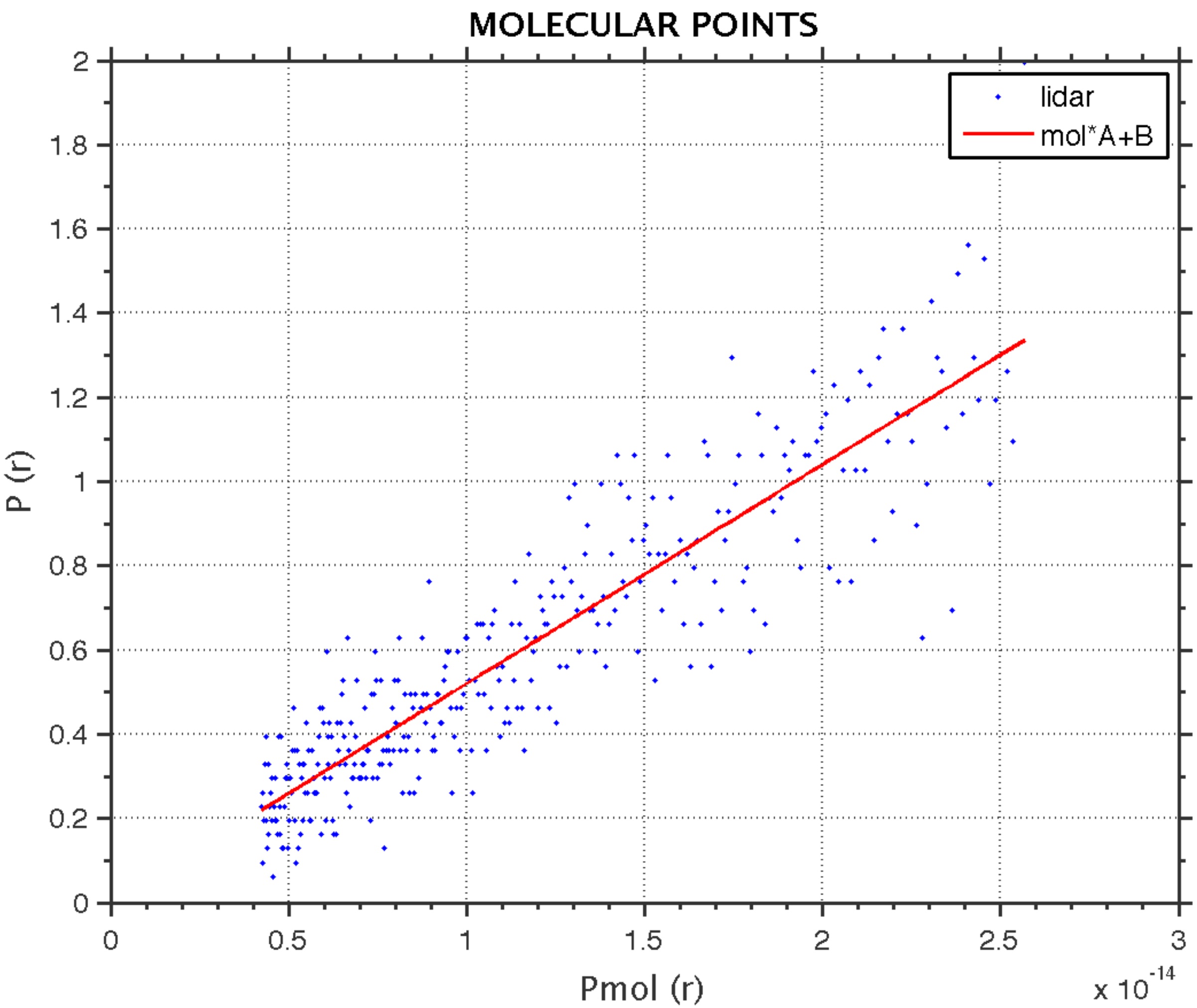
Earlinet, paper #1

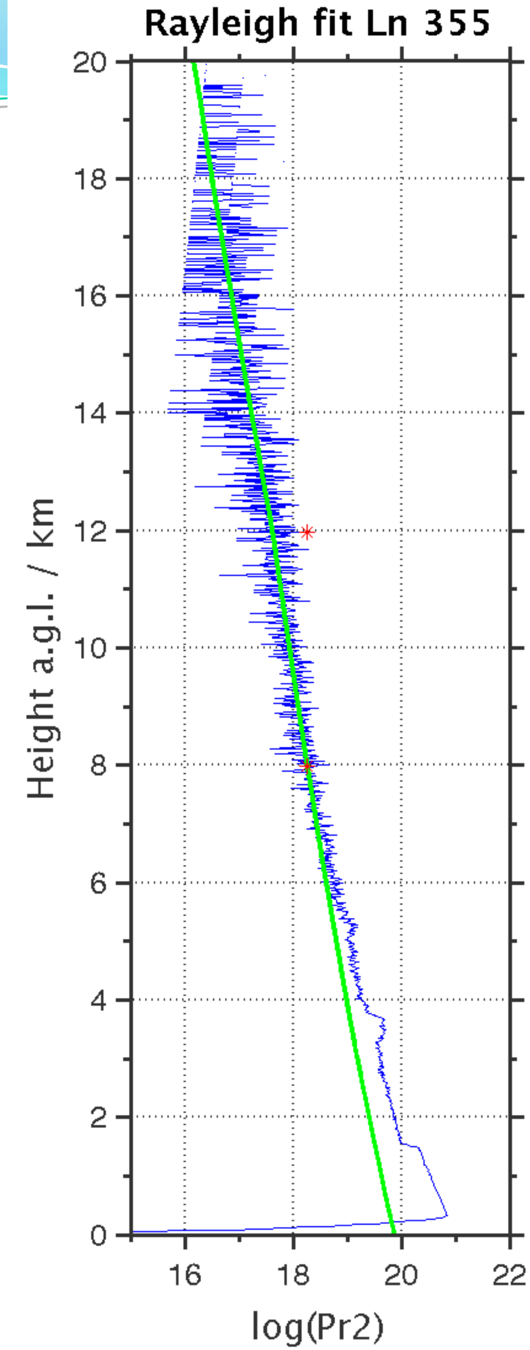
Molecular fit





Molecular fit





Solution to the Lidar equation

- First:
  - We know how EM waves interacts with molecules

$$\beta_{mol} \qquad \alpha_{mol,ext}$$

Only Rayleigh scattering

- Second:
  - We will need to assume something about the aerosol particles

$$\beta_{par} \qquad \alpha_{par,ext}$$

Still 1 equation and 2 unknowns! Impossible to solve unless imposing other constrains.

Solutions to the Lidar equation

- Rewrite the equation as:

$$r^2 P(r,\lambda) = C\beta(r,\lambda)\exp\left[-2\int_0^r \alpha_{ext}(r',\lambda)\,dr'\right]$$

- And consider a new variable:

$$S(r) = \log\left(r^2 P(r,\lambda)\right)$$

- Then

$$S(r) = \log(C) + \log(\beta(r)) - 2\int_0^r \alpha(r',\lambda)\,dr'$$

If homogeneous atmosphere

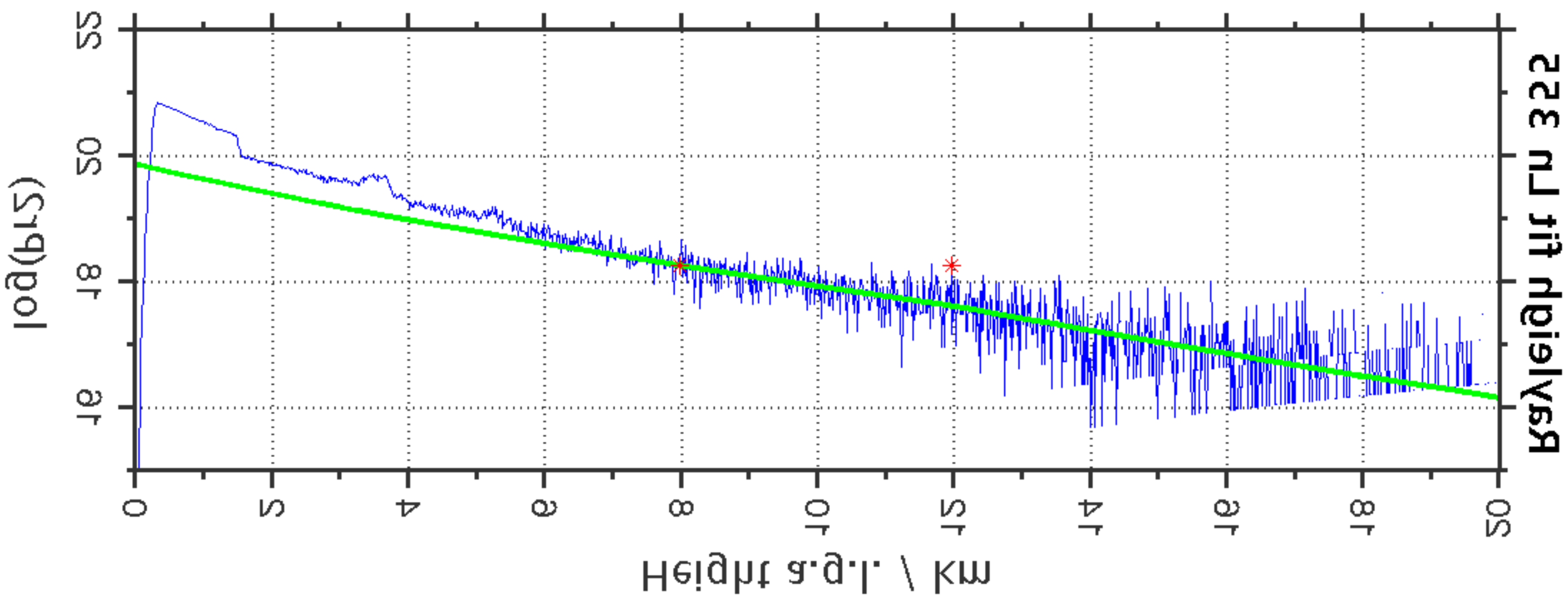
- Taking the derivative to **r**

$$\frac{dS}{dr} = \frac{1}{\beta}\frac{d\beta}{dr} - 2\alpha$$

- If the atmosphere is homogenous, **β = cte**, then

$$\alpha = -\frac{1}{2}\frac{dS}{dr}$$

- We just need a linear-fit where S(r) is a straight line. This is the **slope method**.



log(Pr2)
Height a.g.l. / km
0
2
4
6
8
10
12
14
16
18
20
16
18
20
22

Analytical methods

Hitschfeld & Bordan, J. Meteo. 1954 radar

- Fernald, Ap. Opt. 1972 Forward $\beta = B\alpha$
- Klett, Ap. Opt. 1981 Backward $\beta = B\alpha^{K}$
  - Turbid, $\alpha_p >> \alpha_m \sim 0$
- Fernald, Ap. Opt. 1984 $\beta_m = B_m\alpha_m$ $\beta_p = B_p\alpha_p$
  - Turbid, $\alpha_p \sim \alpha_m > 0$
- Klett, Ap. Opt. 1985 $\beta = B(r)\alpha^{K}$
- Sasano et al, Ap. Opt. 1985 $\beta_m = B_m\alpha_m$ $\beta_p = B_p(r)\alpha_p$

Other methods

Slope method

- Collis, QJRMS 1966
- Viezee et al, JAM 1969

Inverse modeling

- Kastner, 1987
- Yee, 1989

Total Integrated backscatter

Klett-Fernald-Sazano

- Following Ansmann & Muller (Weitkamp, chap 4):

$$\beta_{\text{aer}}(R) + \beta_{\text{mol}}(R) = \frac{S(R)\exp\left\{-2\int_{R_0}^{R}\left[L_{\text{aer}}(r) - L_{\text{mol}}\right]\beta_{\text{mol}}(r)\,\mathrm{d}r\right\}}{\dfrac{S(R_0)}{\beta_{\text{aer}}(R_0) + \beta_{\text{mol}}(R_0)} - 2\displaystyle\int_{R_0}^{R} L_{\text{aer}}(r)S(r)T(r, R_0)\,\mathrm{d}r}$$

- Where:

$$T(r, R_0) = \exp\left\{-2\int_{R_0}^{r}\left[L_{\text{aer}}(r') - L_{\text{mol}}\right]\beta_{\text{mol}}(r')\,\mathrm{d}r'\right\}$$

$$L_{\text{aer}}(R) = \frac{\alpha_{\text{aer}}(R)}{\beta_{\text{aer}}(R)} \qquad L_{\text{mol}} = \frac{\alpha_{\text{mol}}(R)}{\beta_{\text{mol}}(R)}$$